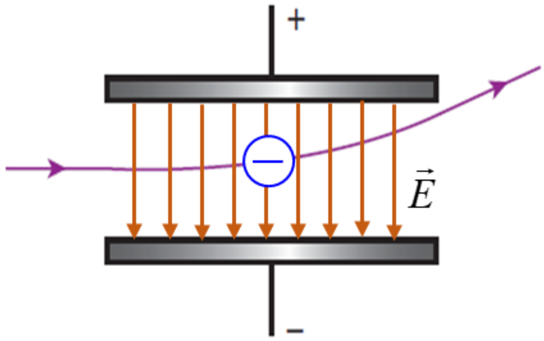
VISUAL PHYSICS ONLINE

MODULE 6

ELECTROMAGNETISM



PARTICLE MOTION IN UNIFORM GRAVITATIONAL and ELECTRIC FIELDS

A frame of reference

Observer

Origin O(0, 0, 0)

Cartesian coordinate axes

(X, Y, Z)

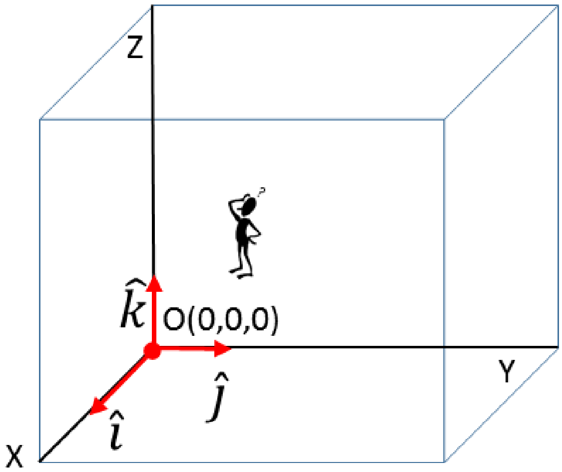
Unit vectors $\hat{i}\ \hat{j}\ \hat{k}$

Specify the units

Define zero point for

potential energy and

electric potential



Note: the magnitude of a vector is always a positive number.

Newton's 2nd Law $\vec{a} = \dfrac{1}{m}\sum \vec{F}$

Electric force $\vec{F}_E = q\,\vec{E}$

Motion of a particle with a uniform acceleration

$$v = u + a\,t$$
$$s = u\,t + \tfrac{1}{2}a\,t^2$$
$$v^2 = u^2 + 2\,a\,s$$
$$s = \bar{v}\,t \qquad \bar{v} = \frac{u+v}{2}$$

Motion the XY plane with uniform acceleration

Apply the equation for uniform acceleration to the X Motion and Y Motion separately

$$\vec{a} = a_x\;\hat{i} + a_y\;\hat{j}$$
$$\vec{v} = v_x\;\hat{i} + v_y\;\hat{j}$$
$$\vec{s} = s_x\;\hat{i} + s_y\;\hat{j}$$
$$v = \left|\vec{v}\right| = \sqrt{{v_x}^2 + {v_y}^2} \qquad \tan\phi = \frac{v_y}{v_x}$$

Energies

Kinetic energy $K = \tfrac{1}{2}m\,v^2$

Electric potential energy and electric potential (uniform electric field)

$$\vec{E} = -E\;\hat{j} \quad U = q\,E\,s_y$$
$$\Delta V = E\,s_y$$
$$U = q\,\Delta V$$

Work

$$W = F_E\,s_y = -\Delta U = q\,E\,s_y$$
$$W = \Delta K = q\,\Delta V$$

Total energy $E = K + U$

Conservation of energy $\Delta K + \Delta U = 0$

Physics is a ***beautiful*** *subject*. The same laws, principles and equations can often be applied to describe and predict the behaviour of a wide range of physical phenomena. As an example, we will examine the motion of a particle in a uniform gravitational field and the motion of charged particles in uniform electric fields and find that the exactly same laws and equations can be applied to both situations.

Motion in a uniform gravitational field

Near the surface of the Earth, to a good approximation the gravitational force $\vec{F}_G$ acting on a particle of mass m is constant (uniform). We will investigate the motion of a particle launched from the Origin with an initial velocity $\vec{u}$ in the uniform gravitational field near the surface. We can predict the velocity $\vec{v}$, displacement $\vec{s}$ and energies (K, U, E) of the particle at any future time t.

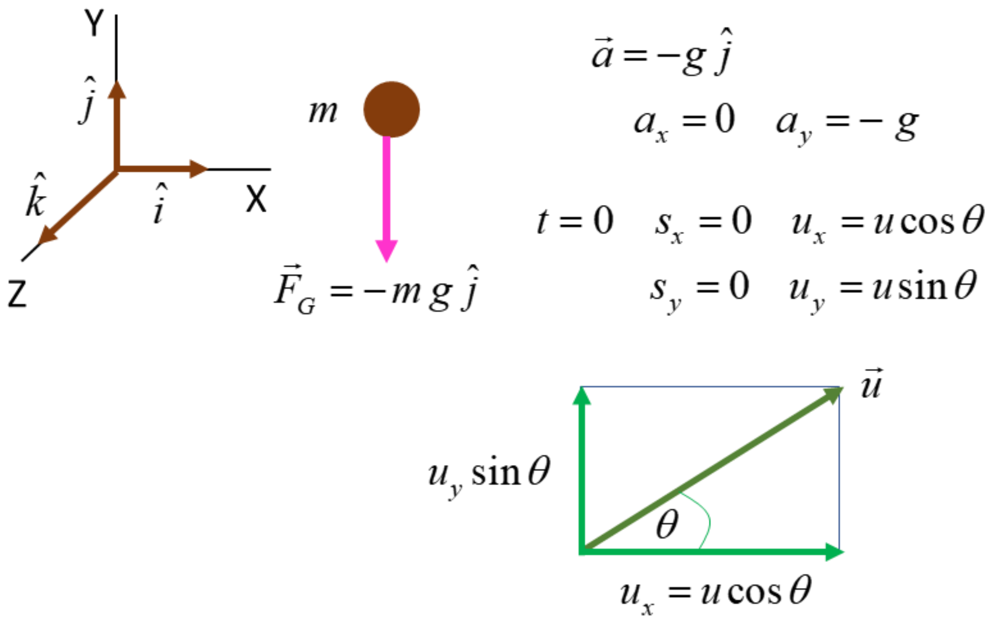


Fig. 1. Frame of reference and force acting on a particle in a uniform gravitational field.

Using Newton's 2nd Law, the acceleration of the particle is constant, because the force acting on the particle is constant

$$\vec{a} = \frac{\vec{F}_G}{m} = -g\,\hat{j} \quad a_x = 0 \quad a_y = -\,g$$

Note: The acceleration of the particle is independent of the particle's mass.

The motion of the particle is described by the equations of uniform accelerated motion which are applied to the motion in the X and Y directions

$$a_x = 0 \quad a_y = -g$$

$$t = 0 \quad s_x = 0 \quad u_x = u\cos\theta$$
$$s_y = 0 \quad u_y = u\sin\theta$$
initial conditions

$$v_x = u_x \quad v_y = u_y + a_y t$$ Graph v_y vs t is a straight line

$$s_x = v_x t \quad s_y = u_y t + \tfrac{1}{2} a_y t^2$$ Graph s_y vs t is a parabola

$$v_y^{\,2} = u_y^{\,2} + 2\, a_y\, s_y$$

$$\bar{v}_y = \frac{u_y + v_y}{2} \quad s_y = \bar{v}_y\, t$$ average velocity

$$|\vec{v}| = v = \sqrt{v_x^{\,2} + v_y^{\,2}} \quad |\vec{s}| = s = \sqrt{s_x^{\,2} + s_y^{\,2}}$$

$$\tan\phi = \frac{v_y}{v_x} \qquad \tan\theta = \frac{s_y}{s_x}$$

$$K = \tfrac{1}{2} m v^2 = \tfrac{1}{2} m v_x^{\,2} + \tfrac{1}{2} m v_y^{\,2}$$ kinetic energy

$$U = m\, g\, s_y \quad s_y = 0 \Rightarrow U = 0$$ gravitational potential energy

$$E = K + U$$ total energy

$$W = \int \vec{F}_G \bullet d\vec{s} = -m\, g\, s_y$$ work done by gravitational force

$$W = \Delta K = \tfrac{1}{2} m v^2 - \tfrac{1}{2} m u^2 = \tfrac{1}{2} m {v_y}^2 - \tfrac{1}{2} m {u_y}^2$$

$$\Delta U = -W = m\, g\, s_y$$

$$\Delta K + \Delta U = 0$$ conservation of energy

1 Predict Observe Explain Exercise

A particle of mass 10 kg is launched with an initial velocity of 10 m.s^{-1} at an angle of 75° w.r.t. the horizontal.

Calculate the time, velocity, position, kinetic energy, potential energy and total energy of the particle at the Origin, when it reaches its maximum height and when it returns to its launch height. Predict the shape of the graphs for:

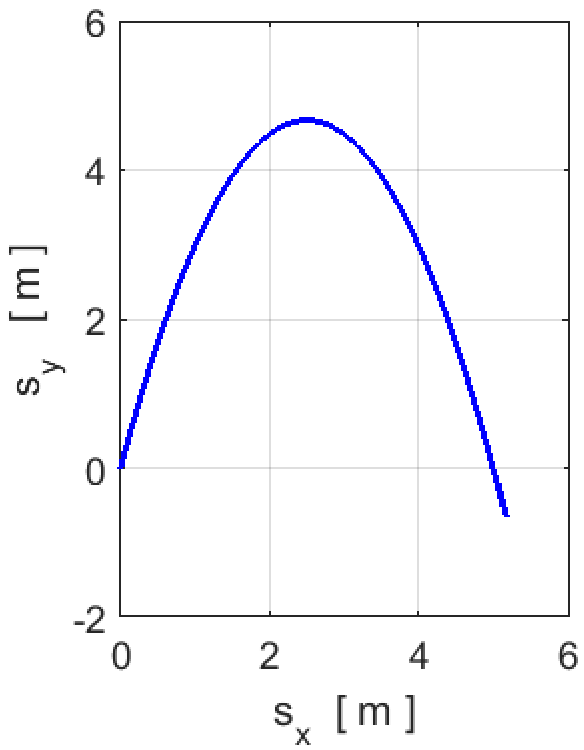
Trajectory (s_x vs s_y)

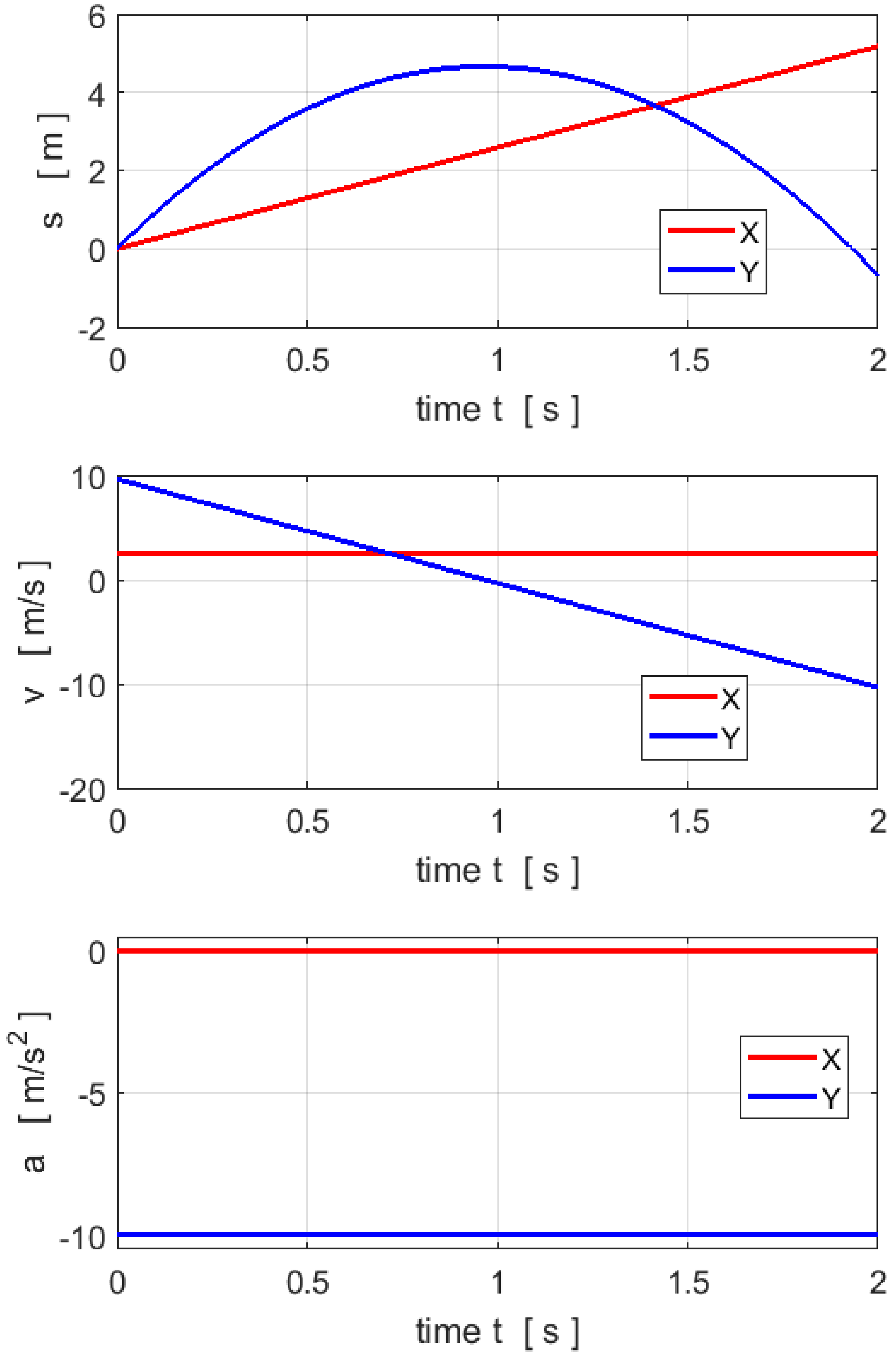
Time graphs $\left(a_x, a_y\right)$ $\left(v_x, v_y\right)$ $\left(s_x, s_y\right)$ $\left(K, U, E\right)$

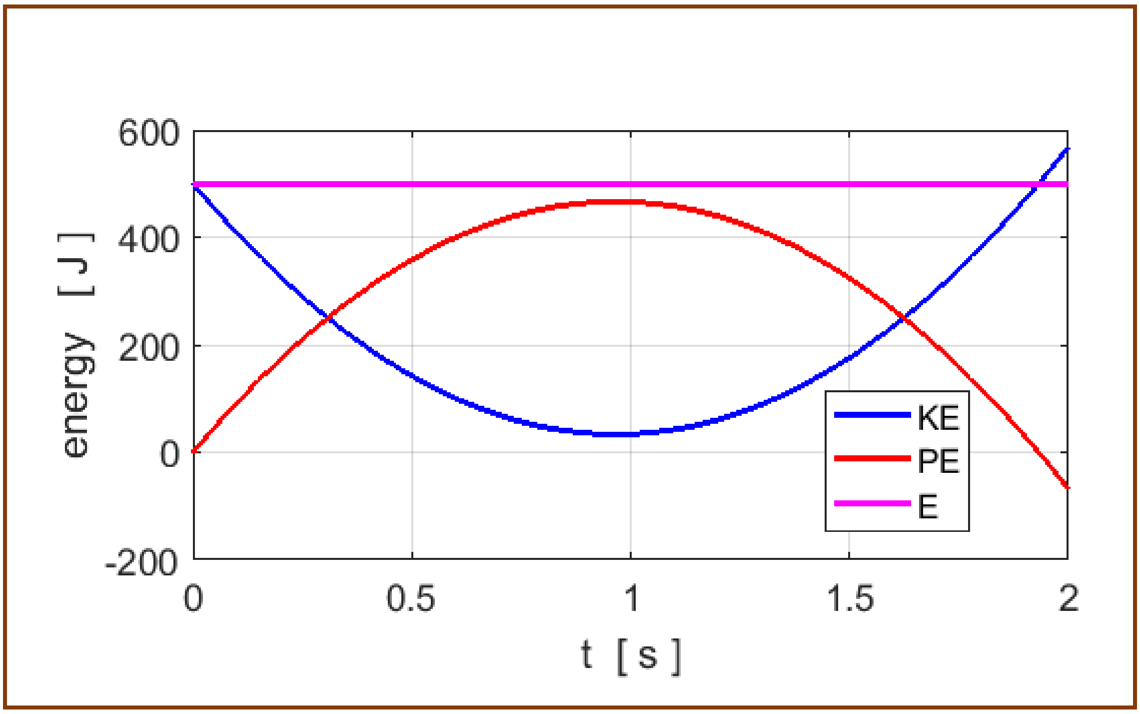
Mark on the graphs, when the particle reaches its maximum height and returns to its launch height.

Solution

Use the graphs shown below to check your calculations and predicted graphs.




s [m]
time t [s]
X
Y
v [m/s]
time t [s]
X
Y
a [m/s2]
time t [s]
X
Y



600
400
200
0
-200
energy [J]
0
0.5
1
1.5
2
t [s]
KE
PE
E

Motion of charged particles in uniform electric fields

How do electric charges move in a uniform electric field?

To get started in answering this question, we will investigate the motion of an electron and a proton in a uniform electric field. The electric field between two oppositely charged parallel plates is approximately a uniform field. As an example, the potential difference between the plates is set at 1000 V and the plate separation is 20 mm. An electron and a proton are released from rest at points mid-way between the two plates. The zero position for the potential energies of the two charges and the potential is at the mid-point between the plates where the two charges are released. The physical situation is shown in figure 2.

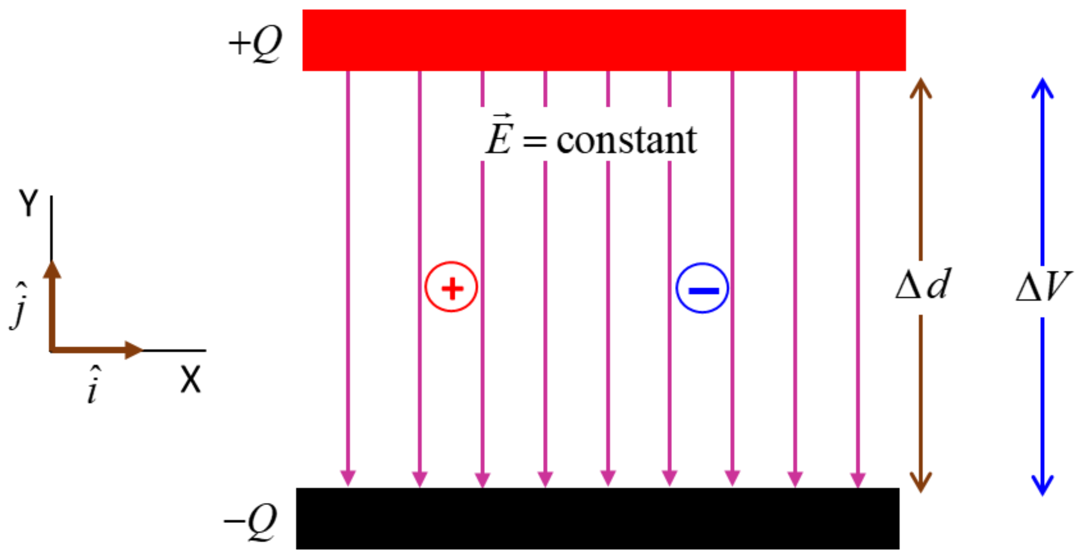


Fig.2. Motion of charged particles in a uniform electric field.

2 Predict Observe Explain Exercise

Predict the following after each charged particle has moved a distance of 5.0 mm from their initial release positions.

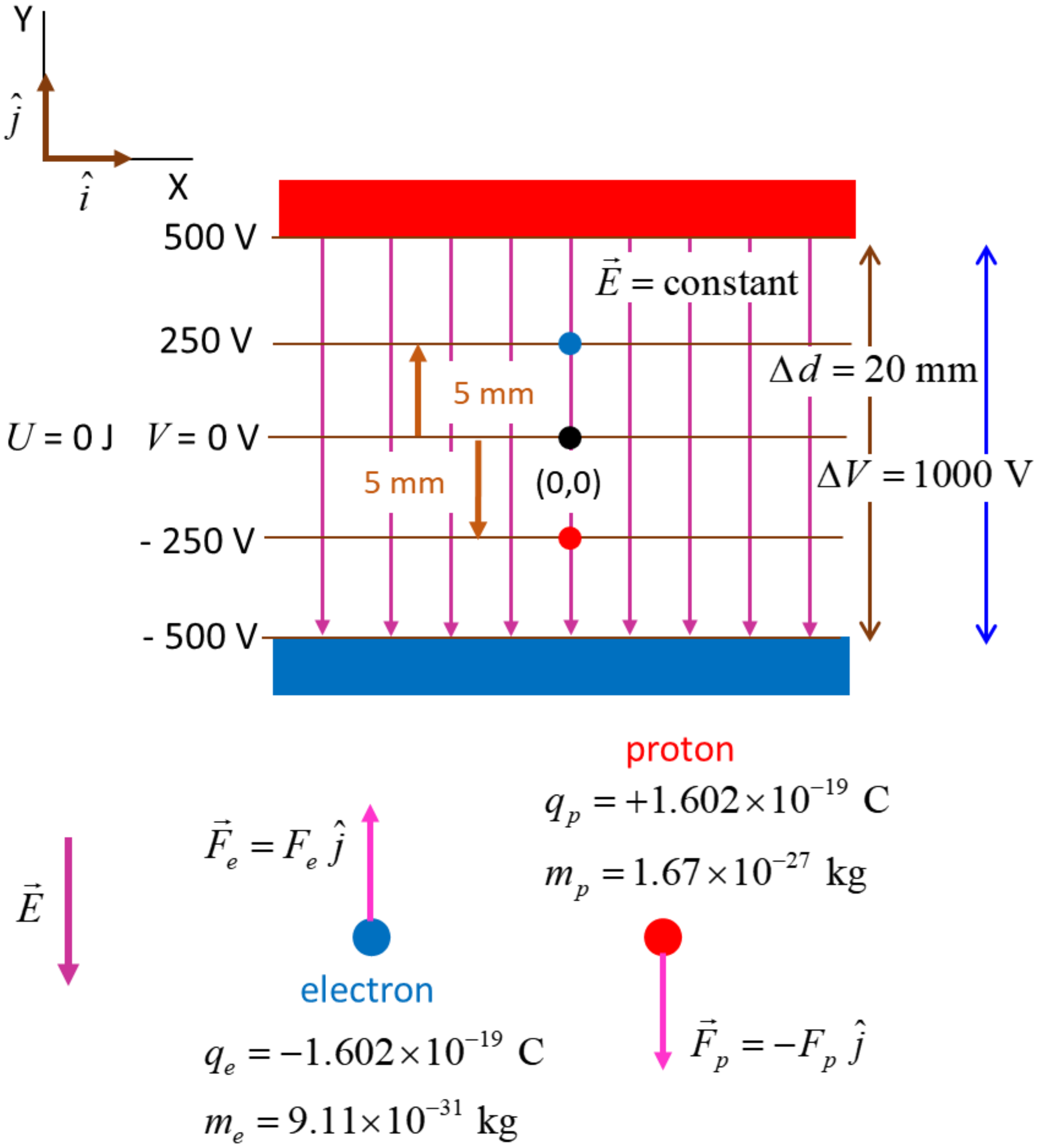
1. The location of each charge.
2. Is the force (magnitude) on the electron less than, equal to or greater than the force on the proton?
3. Is the acceleration (magnitude) on the electron less than, equal to or greater than the acceleration of the proton?
4. The time interval for the electron to travel 5.0 mm is less than, equal to or greater than the time interval for the proton to travel 5.0 mm?
5. Is the work done on the electron by the electric force less than, equal to or greater than the work done on the proton?
6. Is the kinetic energy of the electron less than, equal to or greater than the kinetic energy of the proton?
7. Is the momentum of the electron less than, equal to or greater than the momentum of the proton?
8. Has the potential of the electron decreased, stay the same or increased? Has the potential of the proton decreased, stay the same or increased?
9. Is the potential energy of the electron-field less than, equal to or greater than the potential energy of the proton-field?

Verify each of your predictions by calculating each quantity. Compare your predictions with the numerical results and resolve any discrepancies.

Solution

Visualize the motion of the two charges: The electron will move towards the positive plate as it is attracted to the positive plate and be repelled from the negative plate. The opposite is true for the proton, it will move towards the negative plate.

Annotated diagram including frame of reference



1 Since the charged particles start from rest, they will move in a direction parallel to the electric field lines. So, the distance travelled corresponds to the vertical displacements of the two charges.

Initial position of electron $s_x = 0$ mm $s_y = 0$ mm

Final position of electron $s_x = 0$ mm $s_y = +5.0$ mm

Initial position of proton $s_x = 0$ mm $s_y = 0$ mm

Final position of proton $s_x = 0\ \text{mm} \quad s_y = -5.0\ \text{mm}$

2 The electric force on a charged particle in an electric field is

$$\vec{F} = q\,\vec{E}$$

The magnitudes of the charges on the electron and proton are equal, therefore the magnitudes of the forces on each charged particle are equal.

$$\vec{E} = -E\,\hat{j}$$

$$E = \frac{\Delta V}{\Delta d} = \frac{1000}{20\times10^{-3}}\ \text{V.m}^{-1} = 5.0\times10^{4}\ \text{V.m}^{-1}$$

Electric force on electron and proton

$$F_E = q\,E = \left(1.602\times10^{-19}\right)\left(5.0\times10^{4}\right)\ \text{N} = 8.0\times10^{-15}\ \text{N}$$

Electron $\vec{F}_e = 8.0\times10^{-15}\ \hat{j}\ \text{N}$

Proton $\vec{F}_p = -8.0\times10^{-15}\ \hat{j}\ \text{N}$

3 The acceleration of a particle is given by Newton's Second Law

$$\vec{a} = \frac{1}{m}\sum\vec{F}$$

The mass of the proton is much greater than the mass of the electron. So, the electron's acceleration is much greater than the acceleration of the proton.

$$m_p / m_e = 1.67\times10^{-27} / 9.11\times10^{-31} = 1.8332\times10^{3} \sim 2000$$

Electron $\vec{a}_e = \dfrac{\vec{F}_e}{m_e} = 8.8\times10^{15}\ \hat{j}\ \text{m.s}^{-2}$

Proton $\vec{a}_p = \dfrac{\vec{F}_p}{m_p} = -4.8\times10^{12}\ \hat{j}\ \text{m.s}^{-2}$

4 The acceleration of the electron is greater than the acceleration of the proton. So, the electron will gain speed more quickly and cover a given distance in a shorter time interval than the proton.

Uniform acceleration $s = u\,t + \tfrac{1}{2}a t^2 \quad v = u + a t$

Electron

$$u_e = 0\ \text{m.s}^{-1} \quad s_e = 5.0\times10^{-3}\ \text{m} \quad a_e = 8.8\times10^{15}\ \text{m.s}^{-2} \quad t_e = ?\ \text{s}$$

$$t_e = \sqrt{\frac{2\,s_e}{a_e}} = 1.1\times10^{-9}\ \text{s}$$

$$v_e = a_e\ t_e = 9.4\times10^{6}\ \text{m.s}^{-1}$$

Proton

$$u_p = 0\ \text{m.s}^{-1} \quad s_p = -5.0\times10^{-3}\ \text{m} \quad a_p = -4.8\times10^{12}\ \text{m.s}^{-2} \quad t_p = ?\ \text{s}$$

$$t_p = \sqrt{\frac{2\,s_p}{a_p}} = 4.6\times10^{-8}\ \text{s}$$

$$v_p = a_p\ t_p = 2.2\times10^{5}\ \text{m.s}^{-1}$$

5 Work $W = \int \vec{F}\bullet d\vec{s} \quad W_{net} = \Delta K$

The same work is done on the electron and the proton by the electric force since the magnitude of the displacements are the same.

Electron

$$W_e = F_e\, s_e = q\, E\, s_e = 4.0 \times 10^{-17}\ \text{J}$$

Proton

$$W_p = \left(-F_p\right)\left(-s_p\right) = 4.0 \times 10^{-17}\ \text{J}$$

6 The initial kinetic energies of the electron and proton are both zero, hence, $W = \Delta K = K \quad K = \tfrac{1}{2} m v^2$

The final kinetic energies will be the same since the same amount of work is done on the electron and proton.

Electron

$$K_e = W_e = 4.0 \times 10^{-17}\ \text{J}$$

$$K_e = \tfrac{1}{2} m_e\, v_e{}^2 = 4.0 \times 10^{-17}\ \text{J}$$ alternative calculation

Proton

$$K_p = W_p = 4.0 \times 10^{-17}\ \text{J}$$

$$K_p = \tfrac{1}{2} m_p\, v_p{}^2 = 4.0 \times 10^{-17}\ \text{J}$$ alternative calculation

7 Momentum $\quad p = m\, v = \sqrt{2\, m\, K} \quad K = \dfrac{p^2}{2\, m}$

$$K_e = K_p \quad \frac{p_e^{\;2}}{2m_e} = \frac{p_p^{\;2}}{2m_p} \quad m_e < m_p \quad \Rightarrow \quad p_e < p_p$$

Electron

$$p_e = m_e v_e = 8.5 \times 10^{-24} \text{ kg.m.s}^{-1}$$

Proton

$$p_p = m_p v_p = 3.7 \times 10^{-22} \text{ kg.m.s}^{-1}$$

8 Potential $y = 0 \quad V = 0 \quad \vec{E} = -\frac{dV}{dy}\hat{j} \quad V = -\int E\,dy$

The potential is a linear function of distance for a constant electric field. The potential difference between the plates is 1000 V, therefore the potential of the positive plate is +500 V and the potential of the negative plate is -500 V. The potential at y = 5.0 mm is V = 250 V and the potential at y = - 5.0 mm is V = -250 V. An electron in an electric field moves from a region of lower potential to higher potential. A proton moves from the higher potential region to a lower potential region.

9 Potential energy $y = 0 \quad U = 0$

The changes in potential energy of the electron and the proton are the equal since the work done on the charges is the same.

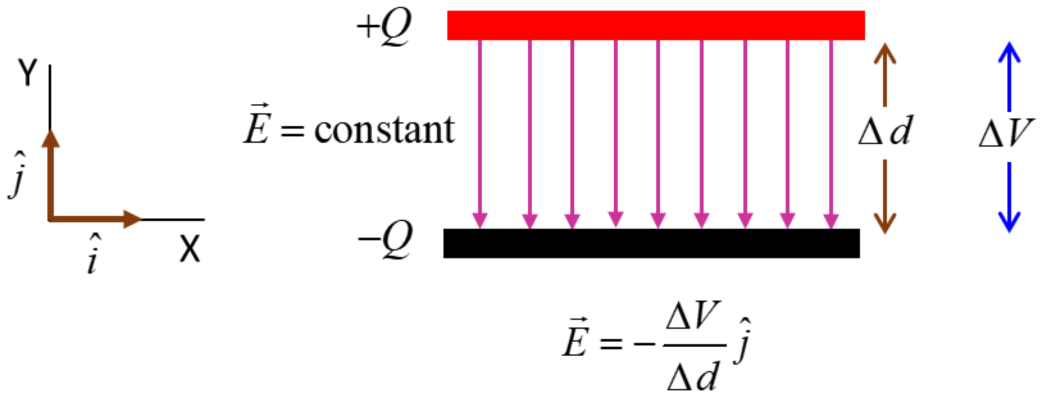
$$\Delta U = U = -W = -4.0\times10^{-17}\ \text{J}$$

Alternatively, energy must be conserved. As the kinetic energy of the electron or proton increases then the potential energy must decrease such that

$$\Delta K + \Delta U = 0 \quad \Delta U = -\Delta K = -4.0\times10^{-17}\ \text{J}$$

We will now consider the [2D] motion of a positive charged particle in a uniform electric field. The electric field is assumed to be uniform in the region between two oppositely charged parallel plates. The potential difference between the two plates is ΔV and the plate separation distance is Δd. So, the uniform electric field between the plates is

$$\vec{E} = -\frac{\Delta V}{\Delta d}\hat{j} \qquad \text{uniform electric field}$$



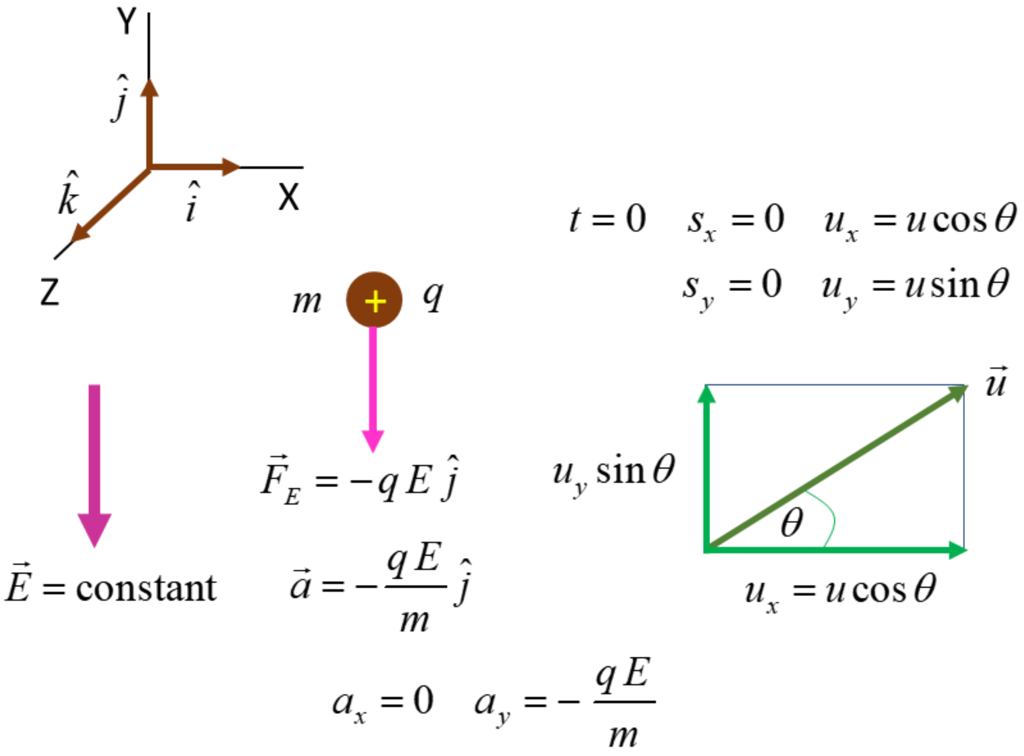


Fig. 3. Projectile motion if a charged particle in a uniform electric field.

The equations describing the motion of a charged particle in a uniform electric field are identical as those describing the motion of an object in the uniform gravitational field near the Earth's surface as given above, except the acceleration of the charged particle is

$$\vec{a} = a_x\, i + a_y\, \hat{j} = -\frac{qE}{m}\hat{j} \quad a_x = 0 \quad a_y = -\frac{qE}{m}$$

For the motion of the charged particle we can also define the very important terms – potential V and potential difference ΔV . The potential and potential difference between two points is related to the difference in potential energy of the electric field and charge.

Electric force acting on charge $\vec{F}_E = -q E \,\hat{j}$

The only force acting on the particle is the electric force. So, the net work done by the electric force is the work done by the electric force which results in the increase in kinetic energy when the charged particle is projected into the uniform electric field. The electric force is a conservative force, therefore, the increase in kinetic energy results in a corresponding decrease in the potential system of the system (particle and charge).

The initial position of the charged particles is taken as the Origin O(0,0) and is the reference point where the potential energy and potential are both zero, $U_0 = 0 \quad V_0 = 0$.

Electric field $\vec{E} = -E\,\hat{j}$

Electric force $\vec{F}_E = -q\,E\,\hat{j} \quad q > 0 \quad E = \left|\vec{E}\right| > 0$

Work done on particle $W = \int \vec{F}_E \bullet d\,\vec{s} = q\,E\,s_y$

Potential energy $\Delta U = -W = -q\,E\,s_y < 0$

Potential $\Delta V = \dfrac{U}{q} = -E\,s_y < 0$

Kinetic energy

$$\Delta K = W = \tfrac{1}{2} m v^2 - \tfrac{1}{2} m u^2 = q\,E\,s_y$$

$$\Delta K = \tfrac{1}{2} m v_x{}^2 + \tfrac{1}{2} m v_y{}^2 - \tfrac{1}{2} m u_x{}^2 - \tfrac{1}{2} m u_y{}^2$$

$$\Delta K = \tfrac{1}{2} m v_y{}^2 - \tfrac{1}{2} m u_y{}^2 \quad \tfrac{1}{2} m v_x{}^2 = \tfrac{1}{2} m u_x{}^2$$

2 Predict Explain Observe Example

A particle of mass 10.0 kg and charge 1.0 mC is launched with an initial velocity of 10.0 m.s^{-1} at an angle of 75^{o} w.r.t. the horizontal in a uniform electric field 1.0x10^{5} V.m^{-1}.

Calculate the time, velocity, position, kinetic energy, potential energy and total energy of the particle at the Origin, when it reaches its position of maximum vertical displacement and when it vertical displacement is again zero. Predict the shape of the graphs for:

Trajectory (s_x vs s_y)

Time graphs $\left(a_x, a_y\right)$ $\left(v_x, v_y\right)$ $\left(s_x, s_y\right)$ $\left(K, U, E\right)$

Mark on the graphs, when the particle reaches its maximum height and returns to its launch height.

Solution

$q = 1.0 \times 10^{-3}\ \text{C} \quad m = 10.0\ \text{kg}$

$u = 10.0\ m.s^{-1} \quad \theta = 75^{o}$

$E = 1.0 \times 10^{5}\ \text{V.m}^{-1}$

$a_x = 0\ \text{m.s}^{-2}$

$$a_y = -\frac{q\,E}{m} = -\frac{\left(1.0 \times 10^{-3}\right)\left(1.0 \times 10^{5}\right)}{10}\ \text{m.s}^{-2} = -10\ \text{m.s}^{-2}$$

The initial velocity and acceleration values are identical to those given in **Example 1**. Therefore, the numerical values and graphs are identical to the results presented in **Example 1**.

In **Example 1**, the acceleration of the particle is independent of the mass – if you launched another particle with the same initial conditions, but with different mass, then all the graphs and numerical values would be the same. This is **not true** for our particle launched in a uniform electric field. The acceleration does dependent upon the particle's mass.

Another important difference between the motion of particles in gravitational fields or electric fields is:

- The gravitational force is always attractive – a particle under the action of only the gravitational force will move from a point of high potential energy towards points of lower potential energy.

- The electric force can be **attractive** or **repulsive**. A positive charge will move from a higher potential point to a lower potential point by the action of the electric force. However, a negative charge will move from a lower potential point to a higher potential point by the action of the electric force.

Consider the case where the vertical acceleration has a magnitude of 5.0 $m.s^{-2}$ instead of 10 $m.s^{-2}$. This could be for the projectile motion of an object on a different planet or the motion of a charged particle in an electric field. Using the results for **Example 1**, how do the trajectory and time to reach maximum height change with the reduced vertical acceleration?

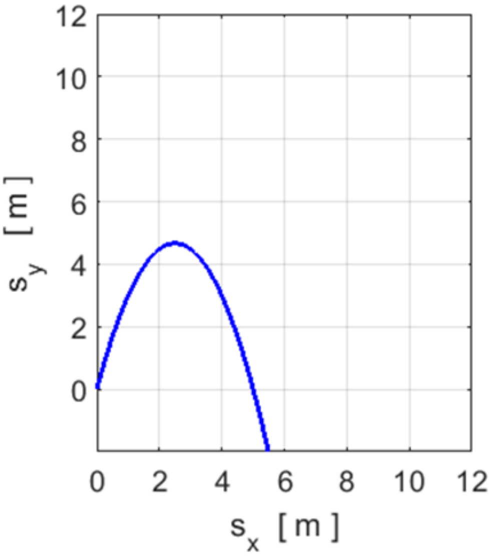
$$|a_y| = 10.0 \text{ m.s}^{-2}$$

time to each max height

$$t = 0.97 \text{ s}$$

max height

$$s_{y\_max} = 4.7 \text{ m}$$



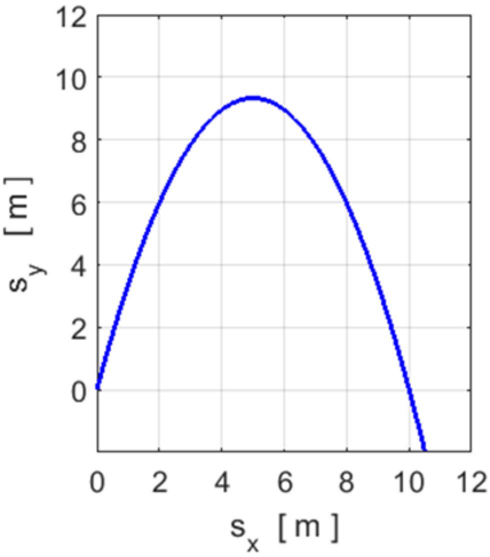
$$|a_y| = 5.0 \text{ m.s}^{-2}$$

time to each max height

$$t = 1.93 \text{ s}$$

max height

$$s_{y\_max} = 9.3 \text{ m}$$



Compare the two sets of results for the vertical acceleration of 5.0 m.s^{-2} and 10.0 m.s^{-2}.

3 Example

A beam of electrons moving in the +X direction with an initial veloicty v_0 enters a region of uniform elctric field $\vec{E}$ directed in the +Y direction generated by a pair of oppositely charged parallel plates (parallel plate capacitor) which has a length x_C in the X direction.

Determine the path of the electron when it is moving through the uniform electric field. What is the vertical distance y_C the electron will be deflected when passing through the uniform electric field.

Descibe the path of the elctron beam after leaving the uniform electric field. The electron beam hits a screen located at a distance x_S from the end of the parallel plate capacitor. What is the vertical deflection y_S of the electron beam from its original trajectory when it strikes the screen?

Solution

Watch Video (concentrate on the motion of charged particles through a uniform electric field).

Review motion with a uniform acceleration

Review [2D] motion in a plane

Think about how to approach the problem by visulaizing the physical situation.

Draw an annoated scientifc diagram.

State the known and unknown physical quantities.

State the equations that you will need.

State the physical principles and concepts needed to solve the problem.

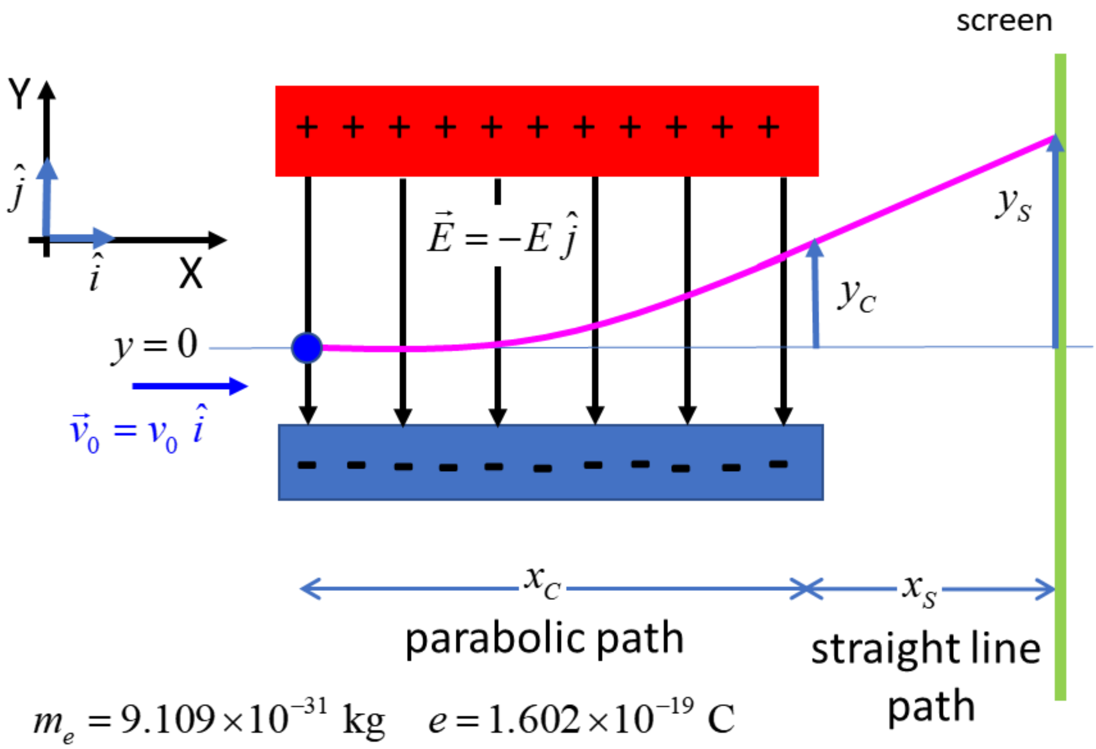
This is a long (difficult ???) problem, but if think about breaking the problem into four smaller problems, then you will find that it is not such a difficult problem.

The problem can be solved using the equations for uniform accelerated motion resolved into the X direction and the Y direction.

$$v = v_0 + a\,t$$

$$s = v_0\,t + \tfrac{1}{2}a\,t^2$$ uniform accelerated motion

$$v^2 = v_0^{\,2} + 2\,a\,s$$



$m_e = 9.109\times10^{-31}$ kg $\quad e = 1.602\times10^{-19}$ C

X motion through capacitor (uniform electric field region)

Initial conditions

$t = 0 \quad v = v_0 \quad a = 0$

Transit time

$s = v_0\,t \quad s = x_C \quad t = ?$

$$t = \frac{x_C}{v_0}$$

Y motion through capacitor (uniform electric field region)

Force acting on electron and its acceleration

$F_E = e\,E = m_e\,a$

$$a = \frac{e\,E}{m_e}$$

Initial conditions

$$t = 0 \quad v = 0 \quad a = \frac{eE}{m_e}$$

Time at which electron leaves electric field region

$$t = \frac{x_C}{v_0}$$

Vertical velocity of electron leaving electric field

$$v = v_0 + at \quad v = \left(\frac{eE}{m_e}\right)\left(\frac{x_C}{v_0}\right) = \frac{eE\,x_C}{m_e\,v_0}$$

Vertical displacement of electron leaving electric field

$$s = v_0\,t + \tfrac{1}{2}a t^2 \quad y_C = \left(\tfrac{1}{2}\right)\left(\frac{eE}{m_e}\right)\left(\frac{x_C}{v_0}\right)^2$$

$$y_C = \frac{eE\,{x_C}^2}{2\,m_e\,{v_0}^2}$$

X motion field free region

Initial conditions

$$t = 0 \quad v = v_0 \quad a = 0$$

Time for electron to travel from capacitor to screen

$$s = v_0\,t \quad s = x_S \quad t = ?$$

$$t = \frac{x_S}{v_0}$$

Y motion field free region

Initial conditions

$$t = 0 \quad v = v_0 = \frac{e\,E\,x_C}{m\,v_0} \quad a = 0$$

Vertical displacment for position of electron striking screen

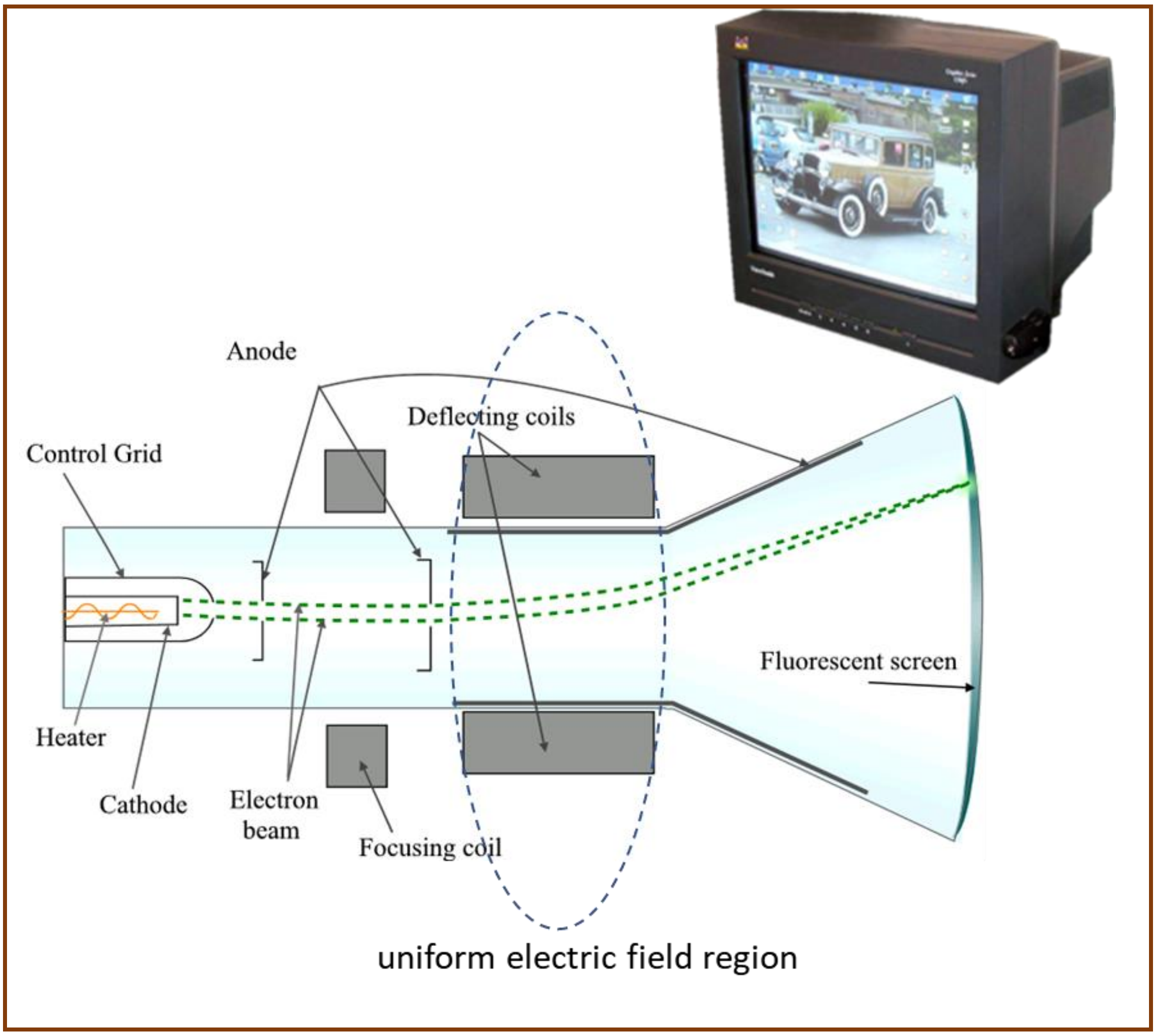
$$y_S = y_C + v\,t$$

$$y_C = \frac{e\,E\,x_C{}^2}{2\,m_e\,v_0{}^2} \quad v = \frac{e\,E\,x_C}{m\,v_0} \quad t = \frac{x_S}{v_0}$$

$$y_S = \frac{e\,E\,x_C{}^2}{2\,m_e\,v_0{}^2} + \left(\frac{e\,E\,x_C}{m_e\,v_0}\right)\left(\frac{x_S}{v_0}\right)$$

$$y_S = \frac{e\,E\left(x_C{}^2 + 2\,x_C\,x_S\right)}{2\,m_e\,v_0{}^2}$$

The vertical deflection y_S of the electron on the screen is proportional to the electric field strength between the plates. In cathode ray tubes used in television sets of the past, the path of the electrons through the tube to the screen could be controlled by changing the electric field between the plates of the capacitors.



[VISUAL PHYSICS ONLINE](http://www.physics.usyd.edu.au/teach_res/hsp/sp/spHome.htm)

http://www.physics.usyd.edu.au/teach_res/hsp/sp/spHome.htm

If you have any feedback, comments, suggestions or corrections please email:

ian.cooper@sydney.edu.au