VISUAL PHYSICS ONLINE

DYNAMICS

ENERGY $E \quad E_K \quad E_P$

WORK W

POWER P

ENERGY [joule J]

Energy is one of the most important concept in Physics. We often refer to kinetic energy, potential energy, gravitational potential energy, stored energy, work, thermal energy, internal energy, heat, heat energy, mechanical energy. sound energy, etc. We know what energy is and we can calculate energy, but, it is **not** possible to give a precise definition of the term energy.

Warning – many of the words used in Physics for example, energy and heat have different meaning to the everyday use of such words.

WORK W and KINETIC ENERGY E_K

When a force acts on an object through a given displacement, energy can be transferred to the object. This transfer of energy is defined as work. The **work done** W by a **single** force acting on an object is

(1) $$W = \int_{\vec{s}_1}^{\vec{s}_2} \vec{F} \bullet d\vec{s} = \int_{\vec{s}_1}^{\vec{s}_2} \left(F\cos\theta\right) d\vec{s}$$ scalar [joule J]

- You will not have to use this formal definition of work, rather you will remember and use a simplified version.
- Work is a **scalar** quantity. When work is done on an object $W > 0$ and when work is done by the object $W < 0$.
- The angle θ is the angle between the two vectors for force and displacement.

  If the force and displacement vectors are **parallel** to each other

  $$\theta = 0^{\circ} \quad \cos\theta = 1 \quad W = \int_{\vec{s}_1}^{\vec{s}_2} F \; d\vec{s}$$

  If the force and displacement vectors are **perpendicular** to each other

  $$\theta = 90^{\circ} \quad \cos\theta = 0 \quad W = 0$$

- If both the **angle** and **force** are constants

$$W = \int_{\vec{s}_1}^{\vec{s}_2} (F\cos\theta)\, ds = (F\cos\theta)\int_{\vec{s}_1}^{\vec{s}_2} ds \qquad \Delta\vec{s} = \vec{s}_2 - \vec{s}_1$$

$$W = (F\cos\theta)\Delta s$$

If the direction of the force and displacement are parallel

$$W = F\,\Delta s$$

- Work is the area under the force vs displacement graph.
- If a number of forces act on the object during the displacement them the **net work** is simply the sum of the work done by each force

$$W_{net} = \sum W$$

Summary

Most general definition of work

(1) $$W = \int_{\vec{s}_1}^{\vec{s}_2} \vec{F}\bullet d\vec{s} = \int_{\vec{s}_1}^{\vec{s}_2} (F\cos\theta)\, d\vec{s}$$

If the force and angle are constants

(2) $$W = (F\cos\theta)\Delta s$$

The force is constant and its direction is parallel to the displacement

(3) $$W = F\,\Delta s$$

The net work is the sum of the work done by each force

(4) $$W_{net} = \sum W$$

Again, consider the action of hitting a tennis ball (mass m) as shown in figure (1). The racket exerts a constant force F on the ball over the distance interval Δs. We assume that the motion is one-dimensional along the X axis. Therefore, we don't need to consider the vector nature of the force and displacement. We can determine the work done W on the tennis ball by the force F applied to the racket during the impact. The constant force F produces a constant acceleration a of the ball which changes its velocity from v_1 to v_2 within the displacement Δs.

Newton's 2nd law $F = m a$

constant a $\quad v_2^{\,2} = v_1^{\,2} + 2 a \Delta s \quad \Delta s = \dfrac{v_2^{\,2} - v_1^{\,2}}{2a}$

work
$$W = F\,\Delta s = (m a)\left(\frac{v_2^{\,2} - v_1^{\,2}}{2a}\right)$$
$$W = \tfrac{1}{2} m v_2^{\,2} - \tfrac{1}{2} m v_1^{\,2}$$

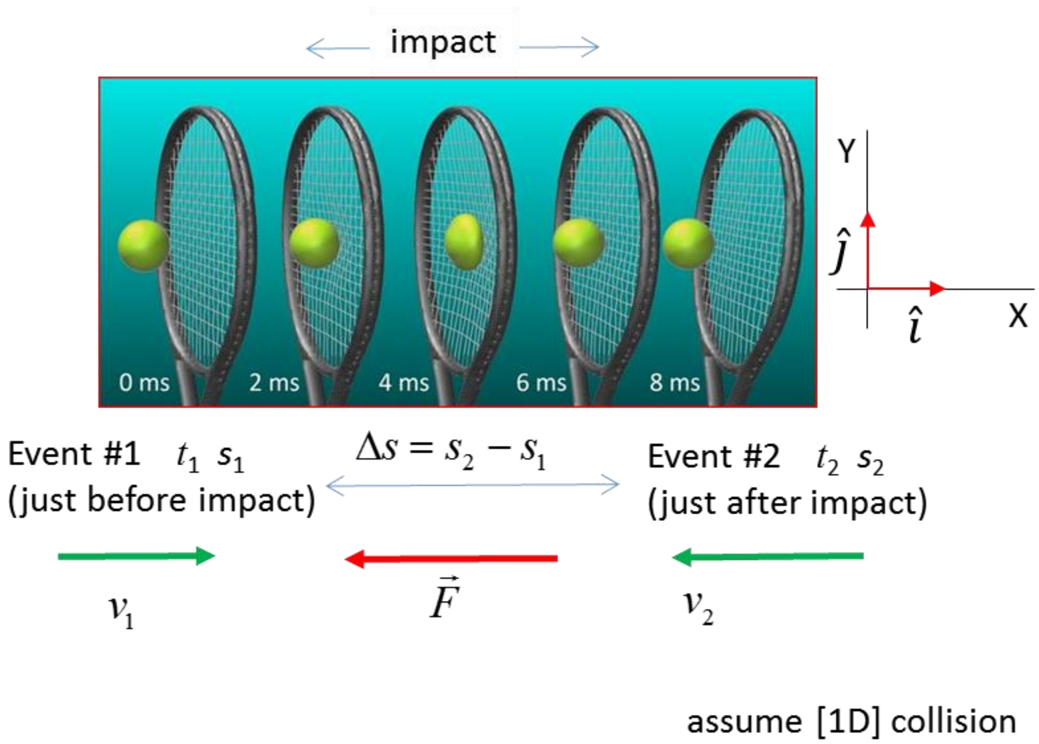


Fig. 1. Work is done on the tennis ball by the racket.

The quantity $\frac{1}{2}mv^2$ is called the **kinetic energy**. It is the energy of a moving object.

(5) $$E_K = \frac{1}{2}mv^2 \qquad \text{scalar } E_K \geq 0 \quad [\text{ J }]$$

The symbol E_K is the best to use for the kinetic energy. Other symbols commonly used include $K \quad KE \quad K.E.$

Work – Energy Theorem

Net Work Done = Change in Kinetic Energy

(6) $$W_{net} = \sum W = \Delta E_K = \tfrac{1}{2} m v_2^{\,2} - \tfrac{1}{2} m v_1^{\,2}$$

The work done by a variable force for [1D] motion is equal to the area under the force vs displacement curve. The area under the F_{avg} rectangle is equal the area under the actual force curve (figure 2).

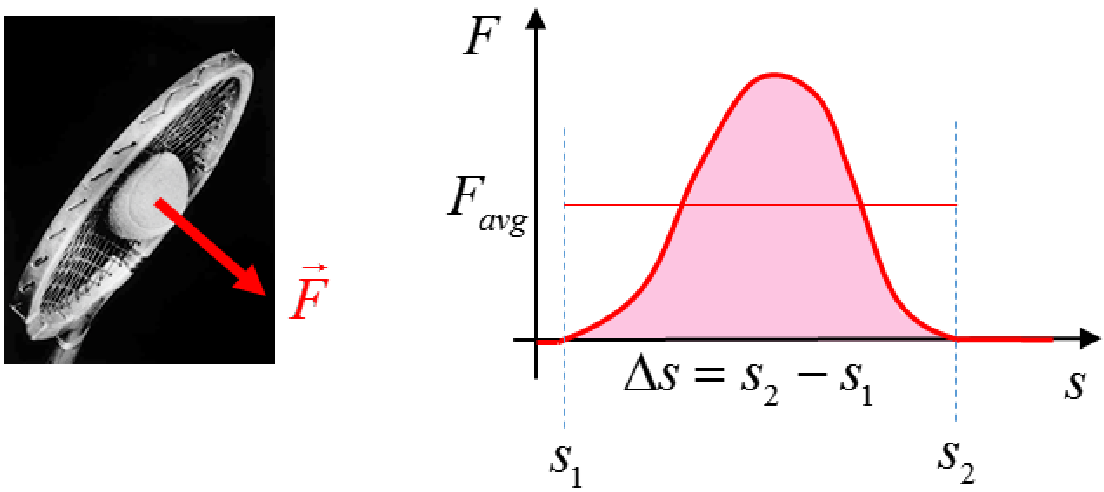


Fig. 2. Work done by a variable force.

$$W = \int_{\vec{s}_1}^{\vec{s}_2} \vec{F} \bullet d\vec{s} = \int_{\vec{s}_1}^{\vec{s}_2} \left(F \cos\theta\right) d\vec{s} = \left(F_{avg} \cos\theta\right) \Delta s = \text{area under } F \,/\, s \text{ curve}$$

POWER P [watt W]

Power is a measure of how quickly energy is transformed. The power associated with the work done is defined as

(7) $P = \dfrac{dW}{dt}$ scalar [watt 1 W = 1 J.s^{-1}]

In terms of average quantities, equation (7) can be expressed as

(8) $P = \dfrac{\Delta W}{\Delta t}$

where the work ΔW is done in the time interval Δt .

For the case in which the force is constant

$$P = \frac{dW}{dt} \quad W = \vec{F} \bullet d\vec{s} \quad \vec{v} = \frac{d\vec{s}}{dt}$$

(9) $P = \vec{F} \bullet d\vec{v}$

The power is proportional to both the force and the velocity.

Once again, the Syllabus documentation contains nonsense statements (see page 36)

work is expressed as $W = \vec{F}_{net}\ \vec{s}$

power is expressed as $P = \vec{F}\, d\vec{v}$

You cannot multiple two vectors. The scalar (dot) product must be used.

Example

To pass a truck safely, a car ($m = 1500\ \mathrm{kg}$) needs to accelerate from 60 km.h^{-1} to 80 km.h^{-1} in less than 3.00 s.

Assuming a constant acceleration, calculate the following for the car in the 3.00 s time interval for the car to overtake safely:

Acceleration of the car

Displacement of the car

Applied force acting on car

Change in kinetic energy of the car

Work done on the car

Average power supplied by the engine

Maximum power supplied by engine

Change in momentum of the car

Impulse given to the car by the engine

Repeat the calculations for the average power when the car accelerates from 80 km.h^{-1} to 100 km.h^{-1} in 3.00 s.

Solution

How to approach the problem

Visualize the physical situation

Problem type: Energy – Work - Power

Identify the car as the System C

Define a frame of reference.

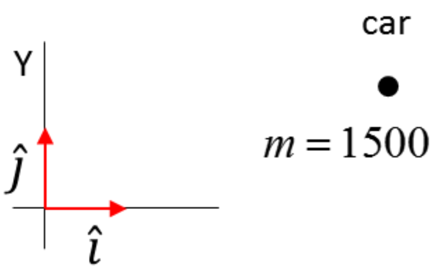
[1D] problem – can treat quantities as scalars

List known & unknown physical quantities (symbols & units)

State physical principles

Annotated scientific diagram

Visualize the physical situation



$m = 1500 \text{ kg}$

Event #1 initial values

Event #2 final values

$v_1 = (60/3.6) \text{ km.h}^{-1}$ $\quad \Delta t = 3.00 \text{ s}$ $\quad v_2 = (80/3.6) \text{ km.h}^{-1}$

$v_1 = 16.67 \text{ m.s}^{-1}$ $\quad v_2 = 22.22 \text{ m.s}^{-1}$

Constant Acceleration $\quad v = u + a\,t$

$$a = \frac{v_2 - v_1}{\Delta t} = \left(\frac{22.22 - 16.67}{3} \right) \text{ m.s}^{-2} = 1.84 \text{ m.s}^{-2}$$

Displacement $s = u\,t + \tfrac{1}{2} a\, t^2$

$$\Delta s = v_1\, \Delta t + \tfrac{1}{2} a\, \Delta t^2 = (16.67)(3) + (0.5)(1.84)(3^2) \text{ m} = 58.38 \text{ m}$$

Force Newton's 2nd Law $\sum \vec{F} = m\,\vec{a}$

$$F = m\,a = (1500)(1.84) \text{ m} = 2.76 \times 10^3 \text{ m}$$

Change in K.E.

$$\Delta E_K = \tfrac{1}{2} m v_2{}^2 - \tfrac{1}{2} m v_1{}^2 = 1.61 \times 10^5 \text{ J}$$

Work done

$$\Delta W = F\, \Delta s = \left(2.76 \times 10^3\right)(58.38) \text{ J} = 1.61 \times 10^5 \text{ J}$$

$\Rightarrow$ Work done = Change in K.E.

Average power

$$P_{avg} = \frac{\Delta W}{\Delta t} = \frac{1.61 \times 10^5}{3} = 5.37 \times 10^4 \text{ W}$$

$$v_{avg} = \frac{v_1 + v_2}{2} = \frac{16.67 + 22.22}{2} \text{ m.s}^{-1} = 19.46 \text{ m.s}^{-1}$$

$$P_{avg} = F\, v_{avg} = \left(2.76 \times 10^3\right)(19.46) = 5.37 \times 10^4 \text{ W}$$

Maximum power

$$P_{\max} = F\,v_2 = \left(2.76\times10^3\right)\left(22.22\right) = 6.13\times10^4 \text{ W}$$

Change in Momentum

$$\vec{p} = m\vec{v}$$

$$\Delta p = m v_2 - m v_1 = (1500)(22.22 - 16.67) = 8.28\times10^3 \text{ kg.ms}^{-1}$$

Impulse

$$\vec{J} = \vec{F}\,\Delta t$$

$$J = \left(2.76\times10^3\right)\left(3\right) \text{ kg.ms}^{-1} = 8.28\times10^3 \text{ kg.ms}^{-1}$$

$\Rightarrow$ Impulse = change in momentum

Acceleration from 80 km.h^{-1} to 100 km.h^{-1}

$$v_1 = \left(80/3.6\right) \text{ km.h}^{-1} = 22.22 \text{ m.s}^{-1}$$

$$v_2 = \left(100/3.6\right) \text{ km.h}^{-1} = 27.78 \text{ m.s}^{-1}$$

$$\Delta W = \tfrac{1}{2} m v_2^{\,2} - \tfrac{1}{2} m v_1^{\,2} = 2.08\times10^5 \text{ J}$$

$$P_{avg} = \frac{\Delta W}{\Delta t} = \frac{2.08\times10^5}{3}\text{W} = 6.94\times10^5 \text{ W}$$

Acceleration from 80 km.h^{-1} to 100 km.h^{-1}

$$P_{avg} = 6.94\times10^4 \text{ W}$$

Acceleration from 60 km.h^{-1} to 80 km.h^{-1}

$$P_{avg} = 5.37\times10^4 \text{ W}$$

As the car gets faster, greater power needs to be supplied by the engine to keep the acceleration constant.

CAR ACCIDENTS

Many young people are killed on our roads each year in car accidents that could have been avoided by driving more slowly. To understand the physics of car accidents and how car manufactures incorporate safety features in their designs it is important to appreciate the concepts of **impulse** and **work**. A common fatal accident is when a car hits a tree or pole. A car of mass m has a velocity u before the collision and is brought to rest ($v = 0$) in a stopping time Δt and stopping distance Δs by the force of the pole acting on the car. This stopping force F depends on the initial momentum or initial kinetic energy of the car.

impulse to stop the car $\quad F\,\Delta t = -m u$

work done to stop the car $\quad F\,\Delta s = -\tfrac{1}{2} m u^2$

- The greater the momentum of the car, the greater the impulse required to stop the car.
- The greater the kinetic energy of the car, the greater the amount of work required to stop the car.

You are more likely to avoid a fatal accident by driving slower.

The **stopping force** is **inversely proportional** to the **stopping time** and **stopping distance**.

If the stopping time and stopping distance are very small as in the case when you smash into a pole the stopping force will be enormous. Figure (3) shows the force vs time and force vs distance for a car of mass 1500 kg initially travelling at 50 km.h^{-1} and 100 km.h^{-1} and being brought to rest by an impact with a pole. The ratio of the velocities is $u_2/u_1 = 2$ but the ratio of stopping distances is $F_2/F_1 = 2^2 = 4$. This is why the front end of modern cars are designed to have a large **crumple zone** to reduce the impact force in a collision. For a stopping distance of 0.20 m, the stopping force for the car travelling at 100 km.h^{-1} is enormous ~3×10^6 N (~200 x weight of car). If the stopping distance was 1.0 m, the force is reduced by a factor of 5.

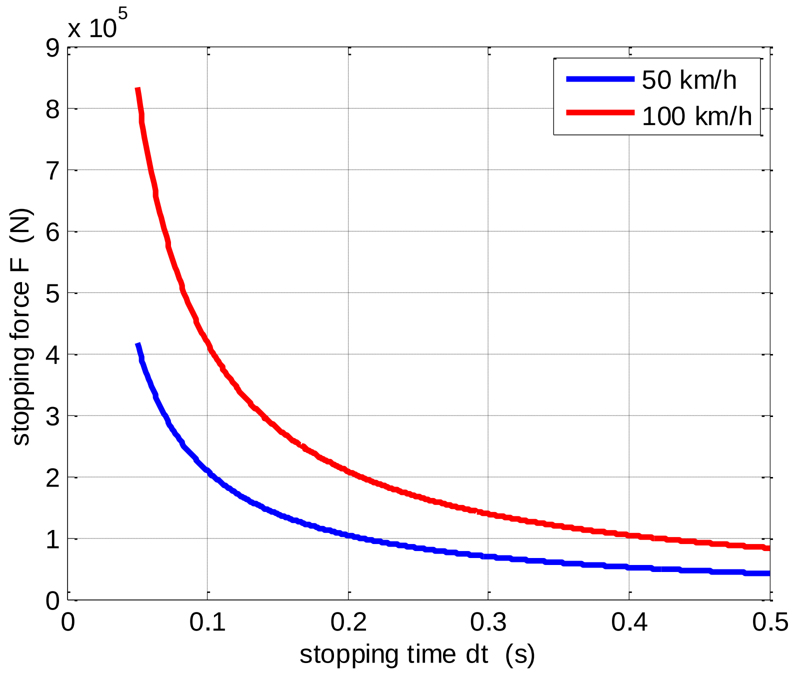


Fig. 3a. Stopping force vs stopping time for a car being brought to rest by an impact with a pole.

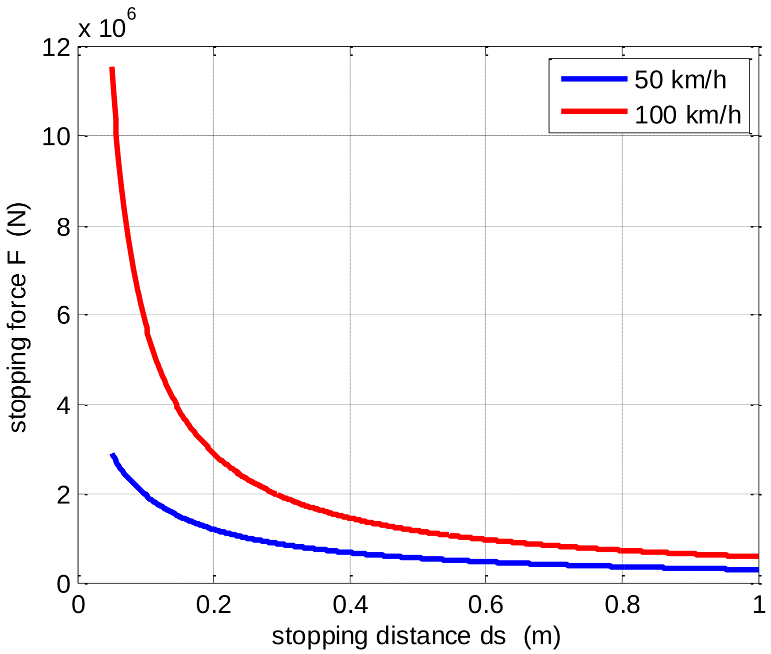


Fig. 3b. Stopping force vs stopping distance for a car being brought to rest by an impact with a pole.

People by law are required to wear seats belts and for cars to achieve a 5 Star safety rating they must have a number of air bags. Seat belts and air bags are designed to increase the stopping time and stopping distance to reduce the force at impact during a collision since

impulse to stop the car $J = F\,\Delta t = F\,\Delta t$ = constant

work to stop the car $W = F\,\Delta s = F\,\Delta s$ = constant

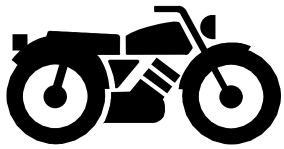
As an example, consider the head a person (m = 5 kg) hitting their head on the steering wheel with the area of contact being 10 mm × 10 mm (10^{-4} m^2). The initial velocity at impact is 36 km.h^{-1} (10 m.s^{-1}) and the stopping distance is 10 mm (0.01 m). The compression strength of bone is 1.7×10^8 N.m^{-2}.

stopping force $F = \dfrac{mu^2}{2\Delta s} = \dfrac{(5)(10^2)}{(2)(0.01)}\,\text{N} = 2.5\times10^4\ \text{N}$

force to smash bone $F_{bone} = (1.7\times10^8)(10^{-4})\,\text{N} = 1.7\times10^4\ \text{N}$

Therefore, in this accident, bones in the skull would certainly be smashed and serious head injuries would occur and most likely death would result. This is the reason cars now are built with collapsible steering columns.

WEB search – use the internet to search for videos of motor bike riders coming off their bikes at high speed.



In watching motor bike racers, it is amazing that a rider comes off their bike at 300 km.h^{-1} and they walk away after the accident? Why?

In an accident, the important thing is not how fast you are travelling but how quickly you stop

PHYSICS AND SPORT

Most sports involve objects colliding during the play - golf, tennis, football, hockey, etc. When objects interact during a short period of time, they may experience very large forces. Evidence of these forces is the distortion in shape of an object at the moment of impact, for example, the distortion of the golf and tennis balls. In the 1970's, the best golfers in the world hit the ball about 250 m.

Today, golfers can hit the ball more than 320 m. *Why?* Manufactures of tennis rackets and golf clubs have applied physical principles and properties of new materials to greatly enhance the momentum imparted to a ball upon impact by the racket or club. From extensive video analysis of sprinters manufactures of shoes now custom make the soles and spikes of top-class runners.

WEB search – use the internet to search design parameters of running shoes and 3D manufacturing.

Why can you damage your feet running barefoot on concrete but not grass?

What are the features of good running shoes?

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If you have any feedback, comments, suggestions or corrections please email:

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