VISUAL PHYSICS ONLINE

DYNAMICS

CONSERVATION OF MOMENTUM AND ENEGY

ELASTIC COLLISIONS

In an elastic collision both momentum and kinetic energy are conserved.

Initial values just before collision (Event #1)

Final values just after collision (Event #2)

Total momentum $\vec{p}_1 = \vec{p}_2$

Total kinetic energy $\frac{1}{2}mv_1^{\,2} = \frac{1}{2}mv_2^{\,2}$

All collisions are inelastic in the real-world. However, for some collisions where objects bounce off each other with little deformation like billiard balls, we can approximate the collision as elastic.

Example billiard ball collisions in [1D] and [2D]

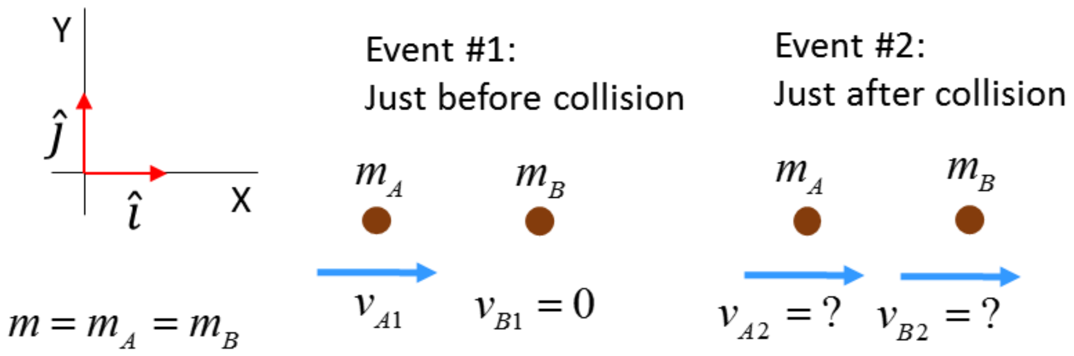
A stationary billiard ball is struck head-on by another billiard ball. Predict the motion of the two billiard balls after the collision.

(1) Two billiard balls travelling at the same speed and in opposite directions collide head-on. Predict the motion of the two billiard balls after the collision.

(2) A stationary billiard ball is stuck a glancing blow by another billiard ball. Show that the two billiard balls move at right angles to each other after the collision

Solution

(1)



Initial momentum and kinetic energy (Event #1: just before collision)

$$p = mv \quad E_K = \tfrac{1}{2} m v^2$$

$$\vec{p}_1 = \vec{p}_{A1} + \vec{p}_{B1} = \left(p_{A1x} + p_{B1x} \right) \hat{i} + \left(p_{B1x} + p_{B1x} \right) \hat{j}$$

$$\vec{p}_1 = m_A v_{A1} \hat{i}$$

$$E_{K1} = \tfrac{1}{2} m_A v_{A1}{}^2$$

Final momentum and kinetic energy (Event #2: just after collision)

$$\vec{p}_2 = \vec{p}_{A2} + \vec{p}_{B2} = \left(p_{A2x} + p_{B2x} \right) \hat{i} + \left(p_{B2x} + p_{B2x} \right) \hat{j}$$

$$\vec{p}_2 = \left(m_A v_{A2} + m_B v_{B2} \right) \hat{i}$$

$$E_{K2} = \tfrac{1}{2} m_A v_{A2}{}^2 + \tfrac{1}{2} m_B v_{B2}{}^2$$

Assume an elastic collision $\Rightarrow$

Conservation of momentum and kinetic energy

(1) $$\vec{p}_1 = \vec{p}_2 \quad m_A = m_B$$
$$v_{A1} = v_{A2} + v_{B2}$$

(2) $$E_{K1} = E_{K2}$$
$$v_{A1}{}^2 = v_{A2}{}^2 + v_{B2}{}^2$$

Solve the two equations for the two unknowns v_{A2} and v_{B2}.

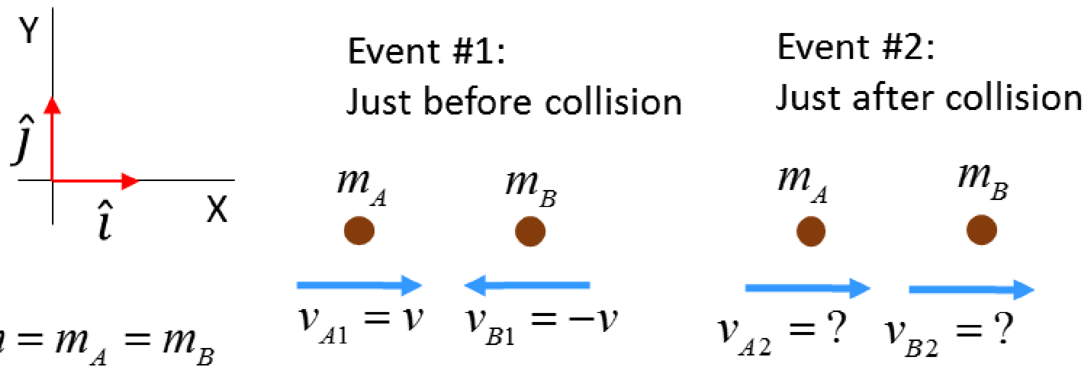
Squaring equation (1) and comparing it with equation (2)

(3) $$v_{A1}{}^2 = v_{A2}{}^2 + v_{B2}{}^2 + 2v_{A2}\,v_{B2}$$

$$v_{A2}\,v_{B2} = 0 \quad \Rightarrow \quad v_{A2} = 0 \quad v_{B2} = v_{A1}$$

After the collision, the incident ball comes to rest, while the target ball moves off with the speed of the incident ball before the collision.

(2)



$$m = m_A = m_B$$

$$v = |v_A| = |v_B|$$

Initial momentum and kinetic energy (Event #1: just before collision)

$$p = mv \quad E_K = \tfrac{1}{2} m v^2$$

$$\vec{p}_1 = \vec{p}_{A1} + \vec{p}_{B1} = \left(p_{A1x} + p_{B1x} \right) \hat{i} + \left(p_{B1x} + p_{B1x} \right) \hat{j}$$
$$\vec{p}_1 = 0$$
$$E_{K1} = \tfrac{1}{2} m_A {v_{A1}}^2 + \tfrac{1}{2} m_B {v_{B1}}^2 = m v^2$$

Final momentum and kinetic energy (Event #2: just after collision)

$$\vec{p}_2 = \vec{p}_{A2} + \vec{p}_{B2} = \left(p_{A2x} + p_{B2x} \right) \hat{i} + \left(p_{B2x} + p_{B2x} \right) \hat{j}$$
$$\vec{p}_2 = \left(m_A v_{A2} + m_B v_{B2} \right) \hat{i}$$
$$E_{K2} = \tfrac{1}{2} m_A {v_{A2}}^2 + \tfrac{1}{2} m_B {v_{B2}}^2$$

Assume an elastic collision $\Rightarrow$

Conservation of momentum and kinetic energy

(1)
$$\vec{p}_1 = \vec{p}_2 \quad m = m_A = m_B$$
$$0 = v_{A2} + v_{B2}$$
$$v_{A2} = -v_{B2}$$

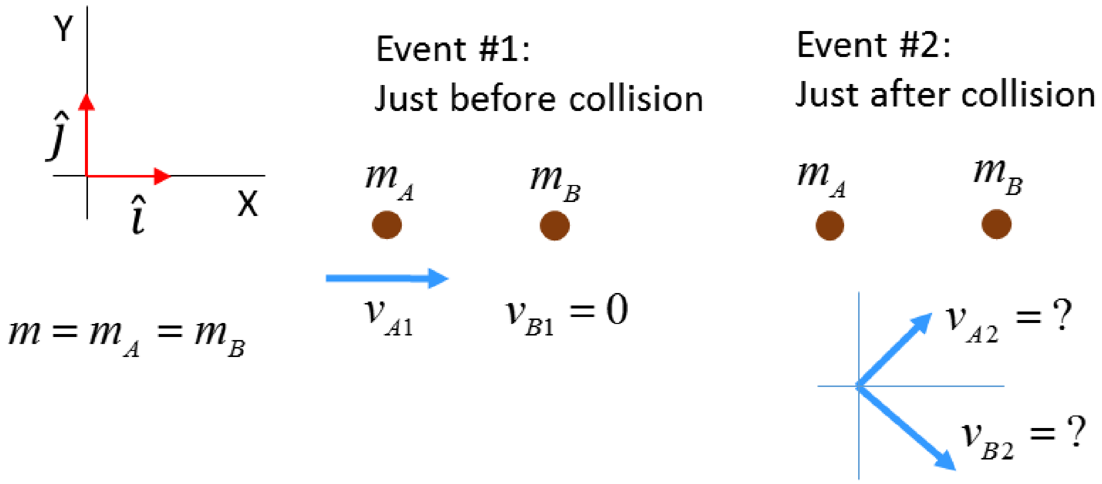
(2)
$$E_{K1} = E_{K2}$$
$$2v^2 = {v_{A2}}^2 + {v_{B2}}^2$$

Solve the two equations for the two unknowns v_{A2} and v_{B2}.

$$v_{A2} = -v \quad v_{B2} = +v$$

After the collision, the balls bounce off each other the at the speeds before the collision but now move in opposite direction.

(3)

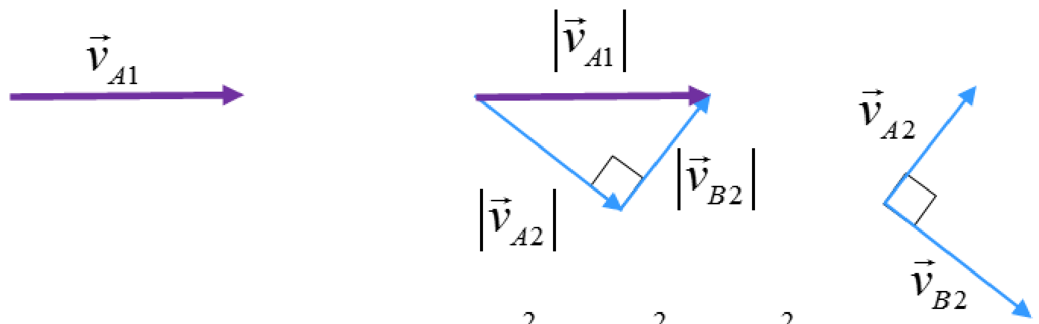


Elastic collision; kinetic energy is conserved

$$E_{K1} = E_{K2}$$

$$v_{A1}^{\ 2} = v_{A2}^{\ 2} + v_{B2}^{\ 2}$$

Geometrically the equation $v_{A1}^{\ 2} = v_{A2}^{\ 2} + v_{B2}^{\ 2}$ is a statement of Pythagoras's Theorem



$$v_{A1}{}^2 = v_{A2}{}^2 + v_{B2}{}^2$$

After the collision, the two balls move at **right angles** to each other.

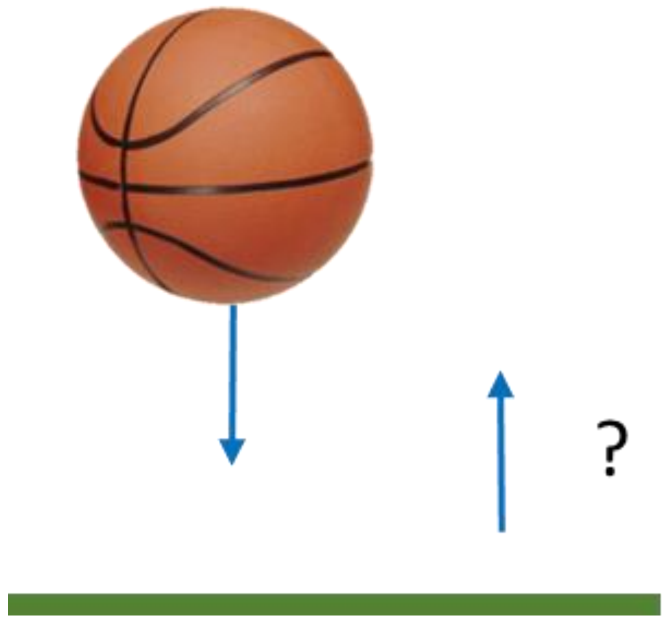
Thinking Exercise

A basketball is dropped and bounces off the ground.

Is momentum conserved?

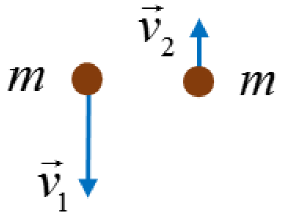
Is energy conserve?

Compare the forces acting on the ball if it bounces or sticks to the ground.



Answer

System: Basketball

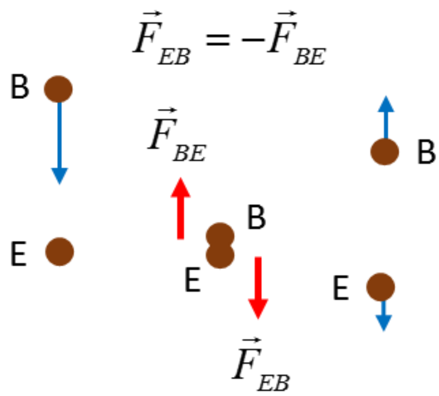


Momentum is **not** conserved since a force acts upon the ball to change its momentum during the time it is in contact with the ground

$$J = F_{avg}\,\Delta t = \Delta p = m\vec{v}_2 - m\vec{v}_1$$

System: Basketball and Earth

System: basketball and Earth



Momentum is conserved

$$\vec{p}_1 = \vec{p}_{B1} + \vec{p}_{E1} = \vec{p}_{B1} + 0 = \vec{p}_{B1}$$

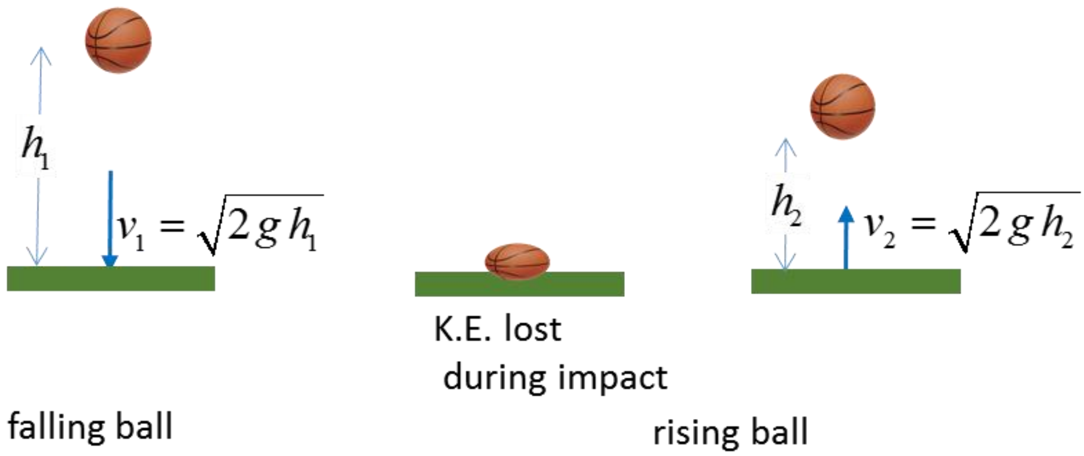
$$\vec{p}_2 = \vec{p}_{B2} + \vec{p}_{E2}$$

$$\vec{p}_1 = \vec{p}_2 \qquad \vec{p}_{B1} = \vec{p}_{B2} + \vec{p}_{E2}$$

Momentum is conserved: after the collision, the Basketball and the Earth move away from each other in opposite directions.

$m_B << m_E$ Earth does not move much

Energy



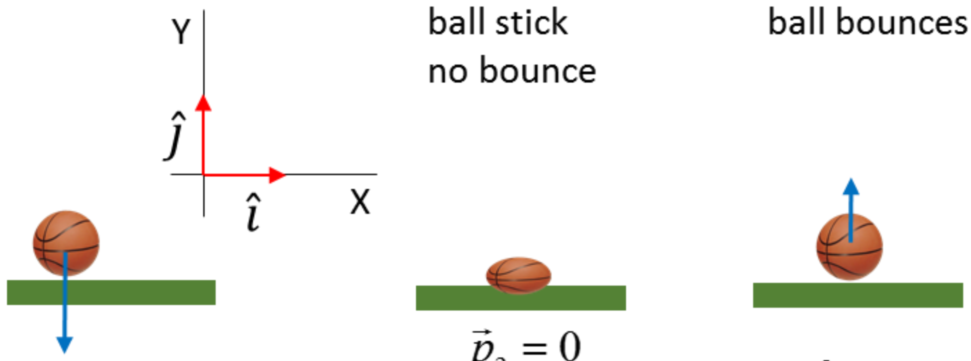
$$E_1 = E_{K1} + E_{P_1} = \text{constant}_1 \qquad E_2 = E_{K2} + E_{P2} = \text{constant}_2$$

$$m g h_1 = \tfrac{1}{2} m v_2^{\ 2} \qquad m g h_2 = \tfrac{1}{2} m v_2^{\ 2}$$

$$E_1 > E_2$$

We can assume mechanical energy (K.E. and P.E.) are conserved when the basketball is in flight. Mechanical energy is dissipated in the impact with the ground so the ball does not bounce as high.

Force



$\vec{p}_2 = 0$

just before impact

$\vec{p}_1 = -mv_1 \; \hat{j}$

$\vec{J}_{stick} = mv_1 \; \hat{j}$

just after impact

$\vec{p}_2 = +mv_2 \; \hat{j}$

Impulse =
change in momentum

$$\vec{J}_{bounce} = m\left(v_1 + v_2\right)\hat{j} > J_{stick}$$

$$\vec{J} = \vec{F}_{avg}\,\Delta t = \Delta\vec{p} = \vec{p}_2 - \vec{p}$$

A **greater force** is exerted by the ground on the ball when it **bounces** compared to no bounce.

Web resources

http://www.animations.physics.unsw.edu.au/jw/momentum.html

http://www.ccpo.odu.edu/~klinck/Reprints/PDF/knightPhysEd75.pdf

https://www.youtube.com/watch?v=yUpiV2I_IRI

https://www.khanacademy.org/science/physics/linear-momentum/elastic-and-inelastic-collisions/v/elastic-and-inelastic-collisions

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If you have any feedback, comments, suggestions or corrections please email:

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