Chapter 3

Currents in unitary symmetry and quark models

3.1 Electromagnetic current in the models of unitary symmetry and of quarks

3.1.1 On magnetic moments of baryons

Main properties of electromagnetic interaction are assumed to be known.

Electromagnetic current of baryons as well as of quarks can be written in a similar way to electrons in the theory with the Dirac equation, only we should account in some way for the non-point-like structure of baryons introducing one more Lorentz structure and two form factors. This current can be deduced from the interaction Lagrangian of the baryon with the electric charge e and described by a spinor $u_B(p)$ and electromagnetic field $A^\mu(x)$ characterized by its polarization vector ϵ^μ:

$$\frac{e}{2M_B(1-\frac{q^2}{4M_B^2})}\bar{u}_B(p_2)[P_\mu G_E(q^2)-$$

$$-i\epsilon_{\mu\nu\rho\sigma}P^\nu q^\rho \gamma^\sigma \gamma_5 G_M(q^2)]u_B(p_1)\epsilon^\mu =$$

$$(\hat{p}-m_B)u_B = 0, \qquad q = p_1 - p_2, \qquad P = p_1 + p_2$$

$$2i\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu],$$

G_E being electric form factor, $G_E(0) = 1$,while G_M is a magnetic form factor and $G_M(0) = \mu_B$ is a total magnetic moment of the baryon in terms of proper magnetons $eh/2m_Bc$. Transition to the model of unitary symmetry means that instead of the spinor u_B written for every baryon one should now put the whole octet B^α_β.

What are properties of electromagnetic current in the unitary symmetry? Let us once more remind Gell-Mann–Nishijima relation between the particle charge Q, 3rd component of the isospin I_3 and hypercharge Y,

$$Q = I_3 + \frac{1}{2}Y.$$

As Q is just the integral over 4th component of electromagnetic current, it means that the electromagnetic current is just a superposition of the 3rd component of isovector current and of the hypercharge current which is isoscalar.

So it can be related to the component $J^1_{\mu 1}$ of the octet of vector currents $J^\alpha_{\mu\beta}$. (More or less in the same way as mass breaking was described by the 33 component of the baryonic current but without specifying its space-time properties.) The part of the current related to the electric charge should assure right values of the baryon charges. Omitting for the moment space-time indices we write

$$eJ^1_1 = e(\bar{B}^\alpha_1 B^1_\alpha - \bar{B}^1_\alpha B^\alpha_1).$$

Here $p = B^1_3$ and so on, are octet baryons with $J^P = \frac{1}{2}^+$. One can see that all the charges of baryons are reproduced .

But the part treating magnetic moments should not coincide in form with that for their charges, as there are anomal magnetic moments in addition to those normal ones. The total magnetic moment is a sum of these two magnetic moments for charged baryons and just equal to anomal one for the neutral baryons.

While constructing baryon current suitable for description of the magnetic moments as a product of baryon and antibaryon octets we use the fact that there are possible as we already know two different tensor structures (which reflects existence of two octets in the expansion $8\times 8 = 1+8+8+10+10^*+27$).

$$J^\alpha_\beta = F(\bar{B}^\gamma_\beta B^\alpha_\gamma - \bar{B}^\alpha_\gamma B^\gamma_\beta) + D(\bar{B}^\gamma_\beta B^\alpha_\gamma + \bar{B}^\alpha_\gamma B^\gamma_\beta) - \frac{2}{3}\delta^\alpha_\beta D\bar{B}^\gamma_\eta B^\eta_\gamma,$$

and trace of the current should be zero, $J^\gamma_\gamma = 0, \alpha, \beta, \gamma, \eta = 1, 2, 3,$. Then electromagnetic current related to magnetic moments (we omit space-time indices here) will have the form

$$J^1_1 = F(\bar{B}^\alpha_1 B^1_\alpha - \bar{B}^1_\alpha B^\alpha_1) + D(\bar{B}^\alpha_1 B^1_\alpha + \bar{B}^1_\alpha B^\alpha_1 - \frac{2}{3}\bar{B}^\alpha_\beta B^\beta_\alpha),$$

wherefrom magnetic moments of the octet baryons read:

$$\begin{array}{ll} \mu(p) = F + \frac{1}{3}D, & \mu(\Sigma^+) = F + \frac{1}{3}D, \\ \mu(n) = -\frac{2}{3}D, & \mu(\Sigma^-) = -F + \frac{1}{3}D, \\ \mu(\Xi^0) = -\frac{2}{3}D, & \mu(\Sigma^0) = \frac{1}{3}D, \\ \mu(\Xi^-) = -F + \frac{1}{3}D & \mu(\Lambda^0) = -\frac{1}{3}D \end{array} \quad (3.1)$$

(Remind that here $B^1_3 = p$.) Agreement with experiment in terms of only F and D constants proves to be rather poor. But many modern model developed for description of the baryon magnetic moments contain these contributions as leading ones to which there are added minor corrections often in the frameworks of very exquisite theories.

Let us put here experimental values of the measured magnetic moments in nucleon magnetons.

$$\begin{array}{rr} \mu(p) = 2.793 & \mu(\Sigma^+) = 2,458 \pm 0.010, \\ \mu(n) = -1.913, & \mu(\Sigma^-) = -1.16 \pm 0.025, \\ & \mu(\Xi^0) = -1.250 \pm 0.014, \\ \mu(\Xi^-) = -0.6507 \pm 0.0025 & \mu(\Lambda^0) = -0.613 \pm 0.04 \end{array} \quad (3.2)$$

And in what way could we construct electromagnetic current of quarks? It is readily written from Dirac electron current:

$$J^{el-m}_\mu = \frac{2}{3}\bar{t}\gamma_\mu t + \frac{2}{3}\bar{c}\gamma_\mu c + [\frac{2}{3}\bar{u}\gamma_\mu u -$$

$$-\frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s] - \frac{1}{3}\bar{b}\gamma_\mu b,$$

and we have put in square brackets electromagnetic current of the 3-flavor model.

And how could we resolve problem of the baryon magnetic moments in the framework of the quark model?

For this purpose we need explicit form of the baryon wave functions with the given 3rd projection of the spin in terms of quark wave functions also with definite 3rd projections of the spin. These wave functions have been given in previous lectures. We assume that in the nonrelativistic limit magnetic moment of the baryon would be a sum of magnetic moments of quarks, while operator of the magnetic moment of the quark q would be $\mu_q \sigma_z^q$ (quark on which acts operator of the magnetic moment is denoted by *). Magnetic moment of proton is obtained as (here $q_1 = q_\uparrow, q_2 = q_\downarrow, q = u, d, s.$)

$$\mu_p = \sum\nolimits_{q=u,d} < p_\uparrow | \hat{\mu}_q \sigma_z^q | p_\uparrow >=$$

$$\frac{1}{6} < 2u_1u_1d_2 - u_1d_1u_2 - d_1u_1u_2 | \hat{\mu}_q \sigma_z^q | 2u_1u_1d_2 - u_1d_1u_2 - d_1u_1u_2 >=$$

$$\frac{1}{6} \sum\nolimits_{q=u,d} < 2u_1u_1d_2 - u_1d_1u_2 - d_1u_1u_2 | \hat{\mu}_q | 2u_1^*u_1d_2 + 2u_1u_1^*d_2 -$$

$$-2u_1u_1d_2^* - u_1^*d_1u_2 - u_1d_1^*u_2 + u_1d_1u_2^* -$$

$$-d_1^*u_1u_2 - d_1u_1^*u_2 + d_1u_1u_2^* >=$$

$$\frac{1}{6}(4\mu_u + 4\mu_u - 4\mu_d + \mu_u + \mu_d - \mu_u + \mu_d + \mu_u - \mu_u) =$$

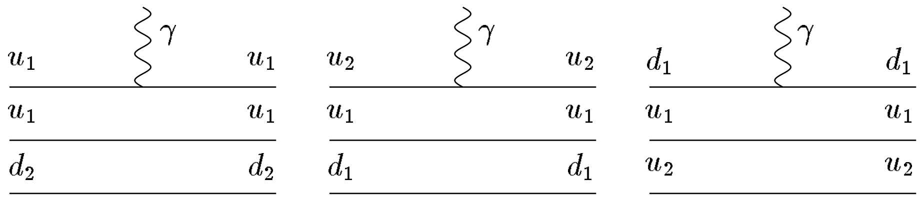
$$\frac{1}{6}(8\mu_u - 2\mu_d) = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d,$$

where we have used an assumption that two of three quarks are always spectators so that

$$< u_1u_1d_2 | \hat{\mu}_q | u_1^*u_1d_2 >=$$

$$=< u_1 | \hat{\mu}_q | u_1^* >= \mu_u \quad etc.$$

Corresponding quark diagrams could be written as (we put only some of them, the rest could be written in the straightforward manner):



In a similar way we can calculate in NRQM magnetic moment of neutron:

$$\mu_n = \sum_{q=u,d} < n_\uparrow|\mu_q \sigma_z^q|n_\uparrow >=$$

$$\frac{1}{6} < 2d_1d_1u_2 - d_1u_1d_2 - u_1d_1d_2|\mu_q\sigma_z^q|2d_1d_1u_2 - d_1u_1d_2 - u_1d_1d_2 >=$$

$$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u,$$

magnetic moment of Σ^+:

$$\mu(\Sigma^+) = \sum_{q=u,s} < \Sigma_\uparrow^+|\mu_q \sigma_z^q|\Sigma_\uparrow^+ >=$$

$$\frac{1}{6} < 2u_1u_1s_2 - u_1s_1u_2 - s_1u_1u_2|\mu_q\sigma_z^q|2u_1u_1s_2 - u_1s_1u_2 - s_1u_1u_2 >=$$

$$\frac{4}{3}\mu_u - \frac{1}{3}\mu_s,$$

magnetic moment of Σ^- :

$$\mu(\Sigma^-) = \sum_{q=d,s} < \Sigma_\uparrow^-|\mu_q \sigma_z^q|\Sigma_\uparrow^- >=$$

$$\frac{1}{6} < 2d_1d_1s_2 - d_1s_1d_2 - s_1d_1d_2|\mu_q\sigma_z^q|2d_1d_1s_2 - d_1s_1d_2 - s_1d_1d_2 >=$$

$$\frac{4}{3}\mu_d - \frac{1}{3}\mu_s,$$

magnetic moment of Ξ^0 :

$$\mu(\Xi^0) = \sum_{q=u,s} < \Xi_\uparrow^0|\mu_q \sigma_z^q|\Xi_\uparrow^0 >=$$

$$\frac{1}{6} < 2s_1s_1u_2 - s_1u_1s_2 - u_1s_1s_2|\mu_q\sigma_z^q|2s_1s_1u_2 - s_1u_1s_2 - u_1s_1s_2 >=$$

$$\frac{4}{3}\mu_s - \frac{1}{3}\mu_u,$$

magnetic moment of Ξ^- :

$$\mu(\Xi^-) = \sum\nolimits_{q=d,s} < \Xi^-_\uparrow|\mu_q\sigma_z^q|\Xi^-_\uparrow >=$$

$$\frac{1}{6} < 2s_1s_1d_2 - s_1d_1s_2 - d_1s_1s_2|\mu_q\sigma_z^q|2s_1s_1d_2 - s_1d_1s_2 - d_1s_1s_2 >=$$

$$\frac{4}{3}\mu_s - \frac{1}{3}\mu_d,$$

and finally magnetic moment of Λ (which we write in some details as it has the wave function of another type):

$$\mu_\Lambda = \sum\nolimits_{q=u,d,s} < \Lambda_\uparrow|\mu_q\sigma_z^q|\Lambda_\uparrow >=$$

$$\frac{1}{4} < u_1s_1d_2 + s_1u_1d_2 - d_1s_1u_2 - s_1d_1u_2|\mu_q\sigma_z^q|$$

$$|u_1s_1d_2 + s_1u_1d_2 - d_1s_1u_2 - s_1d_1u_2 >=$$

$$\frac{1}{4} < u_1s_1d_2 + s_1u_1d_2 - d_1s_1u_2 - s_1d_1u_2|\mu_q|u_1^*s_1d_2 + u_1s_1^*d_2 - u_1s_1d_2^* +$$

$$+s_1^*u_1d_2 + s_1u_1^*d_2 - s_1u_1d_2^*$$

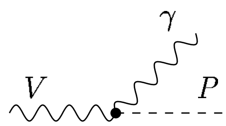
$$-d_1^*s_1u_2 - d_1s_1^*u_2 + d_1s_1u_2^* - s_1^*d_1u_2 - s_1d_1^*u_2 + s_1d_1u_2^* >=$$

$$\frac{1}{4}(\mu_u + \mu_s - \mu_d + \mu_s + \mu_u - \mu_d + \mu_d + \mu_s - \mu_u + \mu_s + \mu_d - \mu_u) = \mu_s,$$

In terms of these three quark magnetons it is possible to adjust magnetic moments of baryons something like up to 10% accuracy.

3.1.2 Radiative decays of vector mesons

Let us look now at radiative decays of vector meson $V \to P + \gamma$.



Let us write in the framework of unitary symmetry model an electromagnetic current describing radiative decays of vector mesons as the 11-component of the octet made from the product of the octet of preusoscalar mesons and nonet of vector mesons:

$$J_1^1 = P_\gamma^1 V_1^\gamma + P_1^\gamma V_\gamma^1 - \frac{2}{3} SpPV =$$

$$2P_1^1 V_1^1 + (P_2^1 V_1^2 + P_3^1 V_1^3) + (P_1^2 V_2^1 + P_1^3 V_3^1) - \frac{2}{3} SpPV =$$

$$2(\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{6}\eta)(\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega) - \frac{2}{3}(\pi^0)\rho^0 + \pi^+\rho^- + \pi^-)\rho^+) + ...$$

We take interaction Lagrangian describing these transitions in the form

$$L = g_{V\to P\gamma} J_{1\mu}^1 A_\mu.$$

Performing product of two matrix and extracting 11 component we obtain for amplitudes of radiative decays in unitary symmetry:

$$M(\rho^0) \to \pi^0\gamma) = (2\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \frac{2}{3}) g_{V\to P\gamma} = \frac{1}{3} g_{V\to P\gamma},$$

$$M(\rho^\pm \to \pi^\pm\gamma) = (1 - \frac{2}{3}) g_{V\to P\gamma} = \frac{1}{3} g_{V\to P\gamma},$$

$$M(\omega^0 \to \pi^0\gamma) = 2\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} g_{V\to P\gamma} = g_{V\to P\gamma} =$$

As masses of ρ and ω^0 mesons are close to each other we neglect a difference in phase space and obtain the following widths of the radiative decays:

$$\Gamma(\omega^0 \to \pi^0\gamma) : \Gamma(\rho^\pm \to \pi^\pm\gamma) = 9 : 1,$$

while experiment yields:

$$(720 \pm 50)Kev : (120 \pm 30)Kev$$

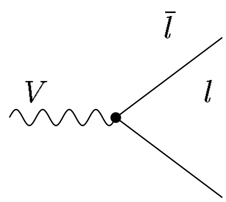
Radiative decay of ϕ meson proves to be prohibited in unitary symmetry,

$$\Gamma(\phi \to \pi^0\gamma) = 0.$$

The experiment shows very strong suppression of this decay, $(6 \pm 0, 6)Kev$.

3.1.3 Leptonic decays of vector mesons $V \to l^+l^-$

Let us now construct in the framework of unitary symmetry model an electromagnetic current describing leptonic decays of vector mesons $V \to l^+l^-$.



In sight of previous discussion it is easily to see that it would be sufficient to extract an octet in the nonet of vector mesons and then take 11-component:

$$J_1^1 = g_{V\bar{l}l}(V_1^1 - \frac{1}{3}V_\alpha^\alpha) = g_{V\bar{l}l}[(\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega) - \frac{1}{3}(\sqrt{2}\omega + \phi)] =$$

$$= g_{V\bar{l}l}(\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{3\sqrt{2}}\omega - \frac{1}{3}\phi)$$

Ratio of leptonic widths is predicted in unitary symmetry as

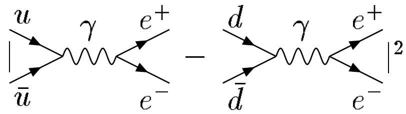
$$\Gamma(\rho^0 \to e^+e^-) : \Gamma(\omega^0 \to e^+e^-) : \Gamma(\phi^0 \to e^+e^-) = 9 \quad : \quad 1 : \quad 2,$$

which agrees well with the experimental data:

$$6, 8Kev \quad : 0, 6Kev \quad : 1, 3Kev \quad .$$

Let us perform calculations of these leptonic decays in quark model upon using quark wave functions of vector mesons For $\rho^0 = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$

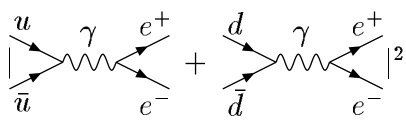
$$\Gamma(\rho^0 \to e^+ + e^-) =$$



$$= \frac{\kappa}{2}[\tfrac{2}{3} - (-\tfrac{1}{3})]^2 = \frac{\kappa}{2}.$$

For $\omega^0 = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$

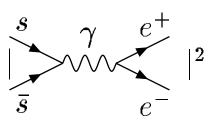
$$\Gamma(\omega^0 \to e^+ + e^-) =$$



$$= \frac{\kappa}{2}[\tfrac{2}{3} + (-\tfrac{1}{3})]^2 = \frac{\kappa}{18}.$$

For $\phi = (\bar{s}s)$

$$\Gamma(\phi^0 \to e^+ + e^-) =$$



$$= \kappa(\tfrac{1}{3})^2 = \frac{\kappa}{2}\frac{2}{9}.$$

It is seen that predictions of unitary symmetry model of the quark model coincide with each other and agree with the experimental data.

3.2 Photon as a gauge field

Up to now we have treated photon at the same level as other particles that as a boson of spin 1 and mass zero. But it turns out that existence of the photon can be thought as effect of local gauge invariance of Lagrangian describing free field of a charged fermion of spin 1/2, let it be electron. Free motion of electron is ruled by Dirac equation $(\partial_\mu \gamma_\mu - m_e)\psi_e(x) = 0$ which could be obtained from Lagrangian

$$L_0 = \bar{\psi}_e(x)\partial_\mu \gamma_\mu \psi_e(x) + m_e \bar{\psi}_e(x)\psi_e(x).$$

This Lagrangian is invariant under gauge transformation

$$\psi'_e(x) = e^{i\alpha}\psi_e(x),$$

α being arbitrary real phase. Let us demand invariance of this Lagrangian under similar but local transformation, that is when α is a function of x:

$$\psi'_e(x) = e^{i\alpha(x)}\psi_e(x),$$

It is easily seen that L_0 is not invariant under such local gauge transformation:

$$L'_0 = \bar{\psi}'_e(x)\partial_\mu \gamma_\mu \psi'_e(x) + m_e \bar{\psi}'_e(x)\psi'_e(x) =$$

$$\bar{\psi}_e(x)\partial_\mu \gamma_\mu \psi_e(x) + i\frac{\partial \alpha(x)}{\partial x_\mu}\bar{\psi}_e(x)\gamma_\mu \psi_e(x) - m_e \bar{\psi}_e(x)\psi_e(x).$$

In order to cancel the term violating gauge invariance let us introduce some vector field A_μ with its own gauge transformation

$$A'_\mu = A_\mu - \frac{1}{e}\frac{\partial \alpha(x)}{\partial x_\mu},$$

and introduce also an interaction of it with electron through Lagrangian

$$ie\bar{\psi}_e(x)\gamma_\mu \psi_e(x)A_\mu,$$

e being coupling constant (or constant of interaction). But we cannot introduce a mass of this field as obviously mass term of a boson field in Lagrangian $m_\gamma A_\mu A^\mu$ is not invariant under chosen gauge transformation for the vector field A_μ. Let us now identify the field A_μ with the electromagnetic field and write the final expression of the Lagrangian invariant under local gauge transformations of the Abelian group $U(1)$

$$L_0 = \bar{\psi}_e(x)\partial_\mu\gamma_\mu\psi_e(x) + ie\bar{\psi}_e(x)\gamma_\mu\psi_e(x)A_\mu$$
$$+m_e\bar{\psi}_e(x)\psi_e(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

where $F_{\mu\nu}$ describe free electromagnetic field

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

satisfying Maxwell equations

$$\partial_\mu F_{\mu\nu} = 0, \quad \partial_\mu A_\mu = 0.$$

The 4-vector potential of the electromagnetic field $A_\mu = (\phi, \quad \vec{A})$ is related to measured on experiment magnetic $\vec{H}$ and electric fields $\vec{E}$ by the relations

$$\vec{H} = rot\vec{A}, \quad \vec{E} = -\frac{1}{c}\frac{\partial\vec{A}}{\partial t} - grad \quad \phi,$$

while tensor of the electromagnetic field $F_{\mu\nu}$ is written in terms of the fields $\vec{E}$ and $\vec{H}$ as

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu = \begin{pmatrix} 0 & E_x & E_x & E_x \\ -E_x & 0 & H_z & -H_y \\ -E_y & -H_z & 0 & H_x \\ -E_z & H_y & -H_x & 0 \end{pmatrix}.$$

Maxwell equations (in vacuum) in the presence of charges and currents read

$$rot\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t}, \quad div\vec{H} = 0,$$
$$rot\vec{H} = \frac{1}{c}\frac{\partial\vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j}, \quad div\vec{E} = 4\pi\rho,$$

where ρ is the density of electrical charge and j is the electric current density. In the 4-dimensional formalism Maxwell equations (in vacuum) in the presence of charges and currents can be written as

$$\partial_\mu F_{\mu\nu} = j_\nu, \qquad j_\nu = (\rho, \quad \vec{j}),$$
$$\epsilon_{\alpha\beta\mu\nu}\partial_\beta F_{\mu\nu} = 0, \quad \partial_\mu A_\mu = 0.$$

3.3 ρ meson as a gauge field

In 1954 that is more than half-century ago Yang and Mills decided to try to obtain also ρ meson as a gauge field. The ρ mesons were only recently discouvered and seemed to serve ideally as quants of strong interaction.

Similar to photon case let us consider Lagrangian of the free nucleon field where nucleon is just isospinor of the group $SU(2)_I$ of isotopic transformations with two components, that is proton (chosen as a state with I_3=+1/2) and neutron (chosen as a state with I_3=-1/2):

$$L_0 = \bar{\psi}_N(x)\partial_\mu\gamma_\mu\psi_N(x) + m_N\bar{\psi}_N(x)\psi_N(x).$$

This Lagrangian is invariant under global gauge transformations in isotopic space

$$\psi'_N(x) = e^{i\vec{\alpha}\vec{\tau}}\psi_N(x),$$

where $\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ are three arbitrary real phases. Let us demand now invariance of the Lagrangian under similar but local gauge transformation in isotopic space when $\vec{\alpha}$ is a function of x:

$$\psi'_N(x) = e^{i\vec{\alpha}(x)\vec{\tau}}\psi_N(x).$$

However as in the previous case The L_0 is not invariant under such local gauge transformation:

$$L'_0 = \bar{\psi}'_N(x)\partial_\mu\gamma_\mu\psi'_N(x) + m_N\bar{\psi}'_N(x)\psi'_N(x) = \bar{\psi}_N(x)\partial_{x_\mu}\gamma_\mu\psi_N(x) -$$

$$+ i\bar{\psi}_N(x)\gamma_\mu\frac{\partial\vec{\tau}\vec{\alpha}(x)}{\partial x_\mu}\psi_N(x) + m_N\bar{\psi}_N(x)\psi_N(x).$$

In order to cancel term breaking gauge invariance let us introduce an isotriplet of vector fields $\vec{\rho}_\mu$ with the gauge transformation

$$\vec{\tau}\vec{\rho'}_\mu = U\vec{\tau}\vec{\rho}_\mu U^\dagger - \frac{1}{g_{NN\rho}}\frac{\partial U}{\partial x_\mu}U^\dagger$$

where $U = e^{i\vec{\alpha}(x)\vec{\tau}}$. An interaction of this isovector vector field could be given by Lagrangian

$$g_{NN\rho}\bar{\psi}_N(x)\gamma_\mu\vec{\tau}\vec{\rho}_\mu\psi_N(x),$$

$g_{NN\rho}$ being coupling constant of nucleons to ρ mesons. Exactly as in the previous case we cannot introduce a mass for this field as in a obvious way mass term in the Lagrangian $m_\gamma \vec{\rho}_\mu \vec{\rho}^\mu$ is not invariant under the chosen gauge transformation of the field $\vec{\rho}_\mu$. Finally let us write Lagrangian invariant under the local gauge transformations of the non-abelian group $SU(2)$:

$$L = \bar{\psi}_N(x)\partial_\mu \gamma_\mu \psi_N(x) + m_N \bar{\psi}_N(x)\psi_N(x) +$$

$$g_{NN\rho}\bar{\psi}_N(x)\gamma_\mu \vec{\tau}\vec{\rho}_\mu \psi_N(x) - \frac{1}{4}\vec{F}^{\mu\nu}\vec{F}_{\mu\nu},$$

$\vec{F}_{\mu\nu}$ describing a free massless isovector field ρ. It is invariant under gauge transformations $U^\dagger \vec{F}'_{\mu\nu} U = \vec{F}_{\mu\nu}$. Let us write a tensor of free ρ meson field $\vec{\tau}\vec{F}_{\mu\nu} = \tau_k F^k_{\mu\nu} \equiv \tilde{F}_{\mu\nu}, \quad ([\tau_i, \tau_j] = 2i\epsilon^{ijk}\tau_k, \quad i,j,k = 1,2,3)$:

$$\vec{F}^k_{\mu\nu} = (\partial_\nu \rho^k_\mu - \partial_\mu \rho^k_\nu) - 2g_{NN\rho} i\epsilon^{kij}\rho^i_\mu \rho^j_\nu$$

or

$$\tilde{F}_{\mu\nu} = (\partial_\nu \tilde{\rho}_\mu - \partial_\mu \tilde{\rho}_\nu) - g_{NN\rho}[\tilde{\rho}_\mu, \tilde{\rho}_\nu]$$

and see that this expression transforms in a covariant way while gauge transformation is performed for the field ρ:

$$U^\dagger(\partial_\nu \tilde{\rho}'_\mu - \partial_\mu \tilde{\rho}'_\nu)U =$$

$$(\partial_\nu \tilde{\rho}_\mu - \partial_\mu \tilde{\rho}_\nu) + [U^\dagger \partial_\nu U, \tilde{\rho}_\mu] - [U^\dagger \partial_\mu U, \tilde{\rho}_\nu],$$

$$U^\dagger[\tilde{\rho}'_\mu, \tilde{\rho}'_\nu]U = [\tilde{\rho}_\mu, \tilde{\rho}_\nu] + \frac{1}{g_{NN\rho}}[U^\dagger \partial_\nu U, \tilde{\rho}_\mu] - \frac{1}{g_{NN\rho}}[U^\dagger \partial_\mu U, \tilde{\rho}_\nu].$$

Finally,

$$U^\dagger \vec{F}'_{\mu\nu} U = U^\dagger(\partial_\nu \tilde{\rho}'_\mu - \partial_\mu \tilde{\rho}'_\nu - g_{NN\rho}[\tilde{\rho}'_\mu, \tilde{\rho}'_\nu])U =$$

$$\partial_\nu \tilde{\rho}_\mu - \partial_\mu \tilde{\rho}_\nu + g_{NN\rho}[\tilde{\rho}_\mu, \tilde{\rho}_\nu] = \vec{F}_{\mu\nu}.$$

It is important to know the particular characteristic of the non-abelian vector field - it proves to be autointeracting, that is, in the term $(-1/4)|\vec{F}^{\mu\nu}|^2$ of the lagrangian new terms appear which are not bilinear in field ρ (as is the case for the abelian electromagnetic field), but trilinear and even quadrilinear in the field ρ, namely, $\rho_\nu \rho_\mu \partial_\nu \rho_\mu$ and $\rho^2_\nu \rho^2_\mu$.

Later this circumstance would prove to be decisive for construction of the non-abelian theory of strong interactions, that is of the quantum chromodynamics (QCD)

The Yang-Mills formalism was generalized to $SU(3)_f$ where the requirement of the local gauge-invariance of the Lagrangian describing octet baryons led to appearance of eight massless vector bosons with the quantum numbers of vector meson octet 1^- known to us.

Unfortunately along this way it proved to be impossible to construct theory of strong interactions with the vector meson as quanta of the strong field. But it was developed a formalism which make it possible to solve this problem not in the space of three flavors with the gauge group $SU(3)_f$ but instead in the space of colors with the gauge group $SU(3)_C$ where quanta of the strong field are just massless vector bosons having new quantum number 'color' named gluons.

3.4 Vector and axial-vector currents in unitary symmetry and quark model

3.4.1 General remarks on weak interaction

Now we consider application of the unitary symmetry model and quark model to the description of weak processes between elementary particles.

Several words on weak interaction. As is well known muons, neutrons and Λ hyperons decay due to weak interaction. We have mentioned muon as leptons (at least nowadays) are pointlike or structureless particle due to all know experiments and coupling constants of them with quanta of different fields act, say, in pure form not obscured by the particle structure as it is in the case of hadrons. The decay of muon to electron and two neutrinos $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ is characterized by Fermi constant $G_F \sim 10^{-5} m_p^{-2}$. Neutron decay into proton , electron and antineutrino called usually neutron $\beta-$ decay is characterized practically by the same coupling constant. However $\beta-$ decays of the Λ hyperon either $\Lambda \to p + e^- + \bar{\nu}_e$ or $\Lambda \to p + \mu^- + \bar{\nu}_\mu$ are characterized by noticibly smaller coupling constant. The same proved to be true for decays of nonstrange pion and strange K meson. Does it mean that weak interaction is not universal in difference from electromagnetic one? It may be so. But maybe it is possible to save universality? It proved to be possible and it was done more than 40 years ago by Nicola Cabibbo by introduction of the angle which naturally bears his name, Cabibbo angle θ_C.

For weak decays of hadrons it is sufficient to assume that weak interactions without change of strangeness are defined not by Fermi constant G_F but instead by $G_F cos\theta_C$ while those with the change of strangeness are defined by the coupling constant $G_F sin\theta_C$. This hypothesis has been brilliantly confirmed during analysis of many weak decays of mesons and barons either conserving or violating strangeness. The value of Cabibbo angle is $\sim 13^o$.

But what is a possible formalism to describe weak interaction? Fermi has answered this question half century ago.

We already know that electromagnetic interaction can be given by interaction Lagrangian of the type current$\times$ field:

$$L = eJ_\mu(x)A^\mu(x) = e\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x).$$

Note that, say, electron or muon scattering on electron is the process of the

2nd order in e. Effectively it is possible to write it the form current$\times$current:

$$L^{(2)} = \frac{e^2}{q^2} J_\mu J^\mu,$$

where q^2 is the square of the momentum transfer. It has turned out that weak decays also are described by the effective Lagrangian of the form current$\times$current but this has been been taken as the 1st order expansion term in Fermi constant:

$$L_W = \frac{G_F}{\sqrt{2}} J_\mu^\dagger J^\mu + \text{Hermitian} \quad \text{Conjugation},$$

The weak current should have the form

$$J_\mu(x) = \bar{\psi}_{\nu_\mu}(x) O_\mu \psi_\mu(x) + \bar{\psi}_{\nu_e}(x) O_\mu \psi_e(x) +$$

$$\bar{\psi}_p(x) O_\mu \psi_n(x) cos\theta_C + \bar{\psi}_p(x) O_\mu \psi_\Lambda(x) sin\theta_C.$$

The isotopic quantum numbers of the current describing neutron β decay is similar to that of the π^- meson while the current describing β decay of Λ is similar to K^- meson. Note that weak currents are charged! Since 1956 it is known that weak interaction does not conserve parity. This is one of the fundamental properties of weak interaction. The structure of the operator O_μ for the charged weak currents has been established from analysis of the many decay angular distributions and turns out to be a linear combination of the vector and axial-vector $O_\mu = \gamma_\mu(1+\gamma_5)$ what named often as $(V-A)$ version of Fermi theory of weak interaction.

Note that axial-vector couplings, those at $\gamma_\mu\gamma_5$ generally speaking are renormalized (attain some factor not equal to 1 which is hardly calculable even nowadays though there is a plenty of theories) while there is no need to renormalize vector currents due to the conservation of vector current, $\partial_\mu V_\mu = 0$.

But dimensional Fermi constant as could be seen comparing it with the electromagnetic process in the 2nd order could be a reflection of existence of very heavy W boson (vector intermediate boson in old terminology) emitted by leptons and hadrons like photon. In this case the observed processes of decays of muon, neutron, hyperons should be processes of the 2nd order in the dimensionless weak coupling constant g_W while $G_F \sim g_W^2/(M_W^2 - q^2)$, and one can safely neglect q^2.

Thus elementary act of interaction with the weak field might be written not in terms of the product *current* $\times$ *current* but instead using as a model electromagnetic interaction:

$$L = \frac{g_W}{\sqrt{2}}(J_\mu W_\mu^+ + J_\mu^\dagger W_\mu^-).$$

3.4.2 Weak currents in unitary symmetry model

And what are properties of weak interaction in unitary symmetry model? As weak currents are charged they could be related to components J_1^2 and J_1^3 of the current octet J_β^α. Comparing octet of the weak currents with the octet of mesons one can see that the chosen components of the current corresponds exactly (in unitary structure, not in space-time one) to π^- and K^- mesons. Vector current is conserved as does electromagnetic current and therefore has the same space-time structure. Then

$$V_{2\mu}^1 cos\theta_C + V_{3\mu}^1 sin\theta_C = (\bar{B}_2^\alpha \gamma_\mu B_\alpha^1 - \bar{B}_\alpha^1 \gamma_\mu B_2^\alpha) cos\theta_C$$

$$+(\bar{B}_3^\alpha \gamma_\mu B_\alpha^1 - \bar{B}_\alpha^1 \gamma_\mu B_3^\alpha) sin\theta_C$$

Here $p = B_3^1$ etc are members of the baryon octet matrix $J^P = \frac{1}{2}^+$. But for the part of currents violating parity similarly to the case of magnetic moments of baryons there are possible two tensor structures and unitary axial-vector current yields

$$-A_{2\mu}^1 = F(\bar{B}_2^\alpha \gamma_\mu \gamma_5 B_\alpha^1 - \bar{B}_\alpha^1 \gamma_\mu \gamma_5 B_2^\alpha) +$$

$$D(\bar{B}_2^\alpha \gamma_\mu \gamma_5 B_\alpha^1 + \bar{B}_\alpha^1 \gamma_\mu \gamma_5 B_2^\alpha)$$

Similar form is true for $A_{3\mu}^1$.

$$A_\mu^+ = A_{2\mu}^1 cos\theta_C + A_{3\mu}^1 sin\theta_C$$

Finally for the neutron β decay one has

$$G_A|_{n \to p+e+\bar{\nu}_e} = (F + D).$$

$$G_{pn}^A = (f + d) cos\theta_C, \quad G_{\Xi^0\Xi^-}^A = (-F + D) cos\theta_C,$$

$$G_{p\Lambda}^A = \frac{1}{\sqrt{6}}(3F + D) sin\theta_C, \quad G_{\Lambda\Xi^-}^A = \frac{1}{\sqrt{6}}(3F - D) sin\theta_C,$$

$$G^A_{n\Sigma^-} = (-F + D)sin\theta_C, \quad G^A_{\Sigma^-\Xi^0} = (F + D)sin\theta_C,$$
$$G^A_{\Lambda\Sigma^-} = \sqrt{\frac{2}{3}}Dcos\theta_C.$$

At $F = 2/3$, $\quad D = 1$ $(SU(6) \supset SU(3)_f \times SU(2)_J)$ $G_A|_{n\to p+e+\bar{\nu}_e} = \frac{5}{3}$. Experimental analysis of all the known leptonic decays of hyperons leads to $F = 0.477 \pm 0,011$, $D = 0.755 \pm 0.011$, which reproduces the experimental result $|G_A/G_V|_{n\to p+e+\bar{\nu}_e}| = 1.261 \pm 0.004$.

3.4.3 Weak currents in a quark model

Let us now construct quark charged weak currents. When neutron decays into proton (and a pair of leptons) in the quark language it means that one of the d quarks of neutron transforms into u quark of proton while remaining two quarks could be seen as spectators. The corresponding weak current yields
$$j^d_\mu = \bar{u}\gamma_\mu(1 + \gamma_5)dcos\theta_C.$$
For the Λ hyperon this discussion is also valid only here it is s quark of the Λ transforms into u quark of proton while remaining two quarks could be seen as spectators. The corresponding weak current yields
$$j^s_\mu = \bar{u}\gamma_\mu(1 + \gamma_5)ssin\theta_C.$$
Logically it comes the form
$$j_\mu = j^d_\mu + j^s_\mu = \bar{u}\gamma_\mu(1 + \gamma_5)d_C,$$
where $d_C = dcos\theta_C + ssin\theta_C$.

Thus in the quark sector (as it says now) left-handed- helicity doublet has arrived: $\begin{pmatrix} u \\ d_C \end{pmatrix}_L = (1 + \gamma_5)\begin{pmatrix} u \\ d_C \end{pmatrix}$. In the leptonic sector of weak interaction it is possible to put into correspondence with this doublet following left-handed-helicity doublets: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ and $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$. (Thus the whole group theory science could be reduced to the group $SU(2)$?) Then it comes naturally an idea about existence of weak isotopic triplet of W bosons which interacts in a weak way with this weak isodoublet:
$$L = g_W\vec{j}_\mu\vec{W}_\mu + H.C.$$

(Let us remain for a moment open a problem of renormalizibility of such theory with massive vector bosons.)

Before we finish with the charged currents let us calculate constant G_A or more exact the ratio G_A/G_V for the neutron β decay in quark model. Nonrelativistic limit for the operator $\gamma_\mu\gamma_5\tau^+$ is $\sigma_z\tau^+$ where τ_+ transforms one of the d quarks of the neutron into u quark.

$$G_A^{np} =< p_\uparrow|\tau_q^+\sigma_z^q|n_\uparrow >=$$

$$= \frac{1}{6} < 2u_1u_1d_2 - u_1d_1u_2 - d_1u_1u_2|\tau^+\sigma_z^q|2d_1d_1u_2 - d_1u_1d_2 - u_1d_1d_2 >=$$

$$\frac{1}{6} < 2u_1u_1d_2 - u_1d_1u_2 - d_1u_1u_2|2u_1^*d_1u_2 + 2d_1u_1^*u_2$$

$$-u_1^*u_1d_2 + d_1u_1u_2^* - u_1u_1^*d_2 > +u_1d_1u_2^* >=$$

$$\frac{1}{6}(-2-2-2-1-2-1) = -\frac{5}{3} \quad (exp. - 1.261 \pm 0.004)$$

Quark model result coincides with that of the exact $SU(6)$ but disaccord with the experimental data that is why in calculations as a rule unitary model of $SU(3)_f$ is used.